# A Guide to Fitting ORGANON's Dynamic Equations 

A System of Equations That Successfully Predicts Expected Tree- and Stand-Level Behavior
by

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#### Abstract

The objective of this paper is to describe the recommended processes for modeling diameter increment, height increment, mortality rate, and crown recession equations using model forms that produce equations which meet biological behavioral expectations. These modeling processes were developed for the ORGANON stand development model in a manner that recognized the historical restrictions on the type and intensity of measurements usually taken on many ownerships (e.g., the data collected in a manner which is restricted to diameter at breast height measured on all trees, total height and height-to-crown-base measured on a subsample of trees, and site index and stand age measured on the plot).

The four equations are designed to predict the behavior of trees growing in untreated stands. In the ORGANON model, treatments are structured as multipliers upon the untreated stand equations, making the accurate modeling of untreated stand development critical to the forest management decision making process.

Described in detail are each model's: rationale for their form, expected predictive behavior, statistical methods used to estimate parameters, and examples of the parameterizations resulting from the application of nonlinear regression fits to four data sets. The final chapter applies one set of model forms and their parameter estimates to show how well the four models work together to predict stand development that meet expected behavior without any adjustments to the basic models.


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## Forward

My expertise on the subject of "growth and yield" (GY) modeling began with my enrollment at the University of Washington in 1973 to start a PhD program in forest biometrics under Professor B. Bruce Bare. I was interested in the relationship between GY models and the operation research (OR) tools used to make decisions on the management of forest stands and aggregate of those stands, and Professor Bare's background and interests matched mine. While at the University of Washington, I continued to work part time doing modeling for the US Forest Service's Forest Survey Project (now FIA) in the Intermountain Forest and Range Experiment Station, which made it possible to self-fund my PhD work. My dissertation topic revolved around the development of a whole-stand/diameter-class GY model for evenaged and unevenaged stands of ponderosa pine (Pinus ponderosa Dougl. ex Laws.) in New Mexico using data from the Fort Rock Experimental Forest in New Mexico, the first experimental forest in the country established by the U.S. Forest Service in 1908. The model was designed to meet the needs of those OR tools used to determine optimal management decisions for both evenaged (e.g., Hann and Brodie 1980) and unevenaged stands (e.g., Hann and Bare 1979), given the constraints imposed by the usage of mainframe computers at that time.

Upon completion of my PhD, I was employed by Oregon State University in 1978 with a 12month faculty position in the Forest Management Department (now the Forest Engineering and Resource Management Department) in which I was to devote 80 percent of my time in order to conduct research in GY modeling and the remainder of my time to graduate instruction in GY Modeling. This heavily research oriented appointment was unique in the Department of Forest Management. This assignment resulted in my developing the ORGANON GY model and then its derivative the CIPSANON GY model, and in my creating and teaching two graduate courses "Forest Modeling" and "Projects in Forest Modeling" and the creation of the undergraduate course "Growth and Yield Modeling", which I latter taught. The undergraduate course was the first in the College of Forestry, and perhaps the country, to integrate the newly constructed personal computer lab (which was run by one of my ex-employees) into the instruction of the class.

The development of the first version of ORGANON occurred in the mixed species (with 27 conifer species and over 17 hardwood species) and complex stand structures found in southwest Oregon (SWO), the most structurally complex forest region in the Pacific Northwest (Franklin and Dyrness 1973). I designed the data collection protocols in a manner that followed sampling procedures used in operational stands (i.e., using a grid of variable radius plots instead of a fixed area plot) in order to minimize measurement error between the sampling procedures used to collect the modeling data and those that would be used in applying the resulting model. I then managed the resulting collection of modeling data on 391 temporary plots in the region that targeted six commercially important conifer species in stands that were being operationally managed using both evenaged and unevenaged practices. The quality and comprehensiveness of data resulting from this data collection effort is unique.

I then developed the model forms and managed the parameter estimation process for the treelevel equations used by SWO-ORGANON to predict:

1. total stem cubic foot volume
2. merchantable stem cubic foot volume
3. Scribner board foot volume
4. stem taper
5. site index (SI) and dominant height growth
6. maximum crown width of open grown trees (MCW)
7. largest crown width of stand grown trees (LCW)
8. crown width profile (CW)
9. H from D
10. HCB and resulting $\triangle \mathrm{HCB}$
11. diameter increment ( $\Delta \mathrm{D}$ )
12. height increment ( $\Delta \mathrm{H}$ )
13. probability of mortality (PM)
14. mean upper $1 / 3$ and mean lower $1 / 3$ of the $\Delta D$ residuals
15. mean upper $1 / 3$ and mean lower $1 / 3$ of the $\Delta H$ residuals

I also developed equations for predicting the maximum size-density trajectory of even-aged stands in the region. I then designed and managed the development of the FORTRAN software programs that used these equations to project stand development over time under various management scenarios. Finally, a comprehensive verification process was conducted to assess how well the set of tree-level equations predicted stand-level development. As a result of this work, I was successful at developing a GY model that performed well in both evenaged and unevenaged stand structures composed of both pure and mixed species.

In a subsequent research project, I expanded the data set in SWO to include plots in old growth stands and in stands with a heavier component of hardwoods than in the original study. Again, I managed the data collection process, designed the model forms and managed the parameter estimation process that used the expanded data set, and I then inserted the revised equations into SWO-ORGANON and conducted another comprehensive verification process.

Ensuing research efforts resulted in the development of the northwest Oregon (NWO), Stand Management Cooperative (SMC), and red alder plantation (RAP) versions of ORGANON. The development of SMC-ORGANON and RAP-ORGANON used data from permanent research plots, which included thinning plots (both SMC and RAP) and fertilization plots (SMC). This led to my developing thinning and fertilization modifier equations for the $\triangle \mathrm{D}, \triangle \mathrm{H}, \mathrm{PM}$, and HCB equations. While all of the modeling data sets in these other versions of ORGANON came from basically evenaged stands, the model forms and resulting parameter estimates developed for these versions of ORGANON closely followed those found in the SWO version of ORGANON. As a result, I believe that the use of these versions to project the development of stand structures that differ from strictly evenaged structure will produce reasonable (not perfect) predictions of tree and stand development.

Long before I was a GY modeler, I was an artist whose medium consisted of both photography and the scale modeling of the logging equipment used in the forest product industry 100 years ago. From these experiences, I relearned the lesson first acquired long ago by artists living in caves in southern Europe: that a recognizable rendition of an object can be created with a few strokes of their "brushes." The art of modeling is creating models that are complex enough to capture the main essence of tree and stand behavior but simple enough to avoid being ensnared in the modeling data set's particular peculiarities. A GY modeler is faced with a limited collection of the type of measurements available for modeling purposes and an extremely small sample of the population in which those measurements are available to characterize the entire population. Therefore, a GY modeler would be wise to learn that success in their avocation will require a combination of skills in both science, to use the data available in a manner that is scientifically justifiable, and in art, in order to use that creative artistic "brush" to reasonably extend those models to areas not currently covered by data.

It would also be useful to a GY modeler if they had the powers of Cassandra (but not her fate). For me, it is clear climate change is causing clashing convictions in the forestry profession and beyond. Some forest activists are arguing for a complete cessation of tree harvesting in order to lock up more carbon (e.g., Buotte et al. 2019), while others are arguing that forest products are vital, sustainable products needed for the wellbeing of human beings and can contribute to carbon storage. Many conifer species in the Pacific Northwest have a culmination of MAI that is considerably longer than species in other regions. Therefore, one compromise could be to grow and then harvest trees at longer rotations than currently used in intensive forest management. In the west, the management of long rotations for evenaged stands will probably also include thinning the stands in order to better fireproof them.

An alternative comprise would be to use uneven-aged management to create stands managed in a way that would produce widely spaced dominants in order to allow frequent ground fires that do not destroy them. Currently, information from research installations about the GY consequences of such scenarios is limited or unavailable. Whether or not one believes these particular scenarios, the point is that any solution will require robust GY models developed for as wide a range of management scenarios as possible and this will result in the need to extend predictive capabilities beyond the GY data currently available.

Ideally, the resulting equations will provide reasonable predictions across the full range of tree sizes and stand conditions, both natural and manmade, that can be experienced by the species of interest. In attempting to meet this goal over the past 45 years, I have made an effort to gather information on the extreme values that tree and stand attributes have achieved by various species. I then keep these "extreme" values in mind when evaluating projected tree and stand development of those species from the GY models I have developed. Examples for Douglas-fir include:

1. I have measured annual $\Delta \mathrm{D}$ as high as 1.8 -inches per year for young plantations and as high as 0.3 -inches per year in 70 " old growth trees.
2. I have measured trees with live crown ratios (CR) that range as low as 0.05 .
3. The historic popular literature contains numerous reports of old growth trees reaching values of D over 180-inches and values of H greater than 300-feet.
4. I have measured a plot in a 229 year old, evenaged stand in southwest Oregon with basal area per acre of 542 square feet per acre, a stand density index of 691, a top height of 209 feet, and containing 36,144 total stem cubic foot volume per acre. Worthington (1958) reported that a one acre plot in a 375 year old, evenaged stand on the Olympic National Forest had a basal area per acre of 714 square feet per acre, a stand density index of 710, a stand height of 285 feet, and containing 52,780 cubic feet per acre. While these large "stand" values are useful to understanding the potential extreme values that "stand" attributes can achieve, it must be remembered that they are plot-level values and, therefore, represent small areas of actual stands.

It should also be remembered that, while the data sets available for modeling $\Delta \mathrm{D}, \Delta \mathrm{H}, \mathrm{PM}$, and $\Delta H C B$ are extremely small in relation to the population in which they will be applied, the data sets used to model attributes such as MCW, LCW, and CW are even smaller. This knowledge has led me to concentrate on developing model forms in a manner that utilizes the data on extremes, such as above, to create equations that will reasonably extrapolate to the broader population than exists in the modeling data.

While the model forms described in the following chapters were developed starting almost forty years ago, the same model forms are now the basis for the equations being created by the Center for Intensive Silviculture (CIPS). The reason I am using the ORGANON equations as my examples in the following chapters is because they are in the public domain and the CIPS equations are proprietary. Furthermore, these equations have been extensively evaluated/validated by a number of large forest landowners in the Pacific Northwest and they have been found to perform better than any of the other existing GY models in the region. For example one, one large company evaluated the system of equations in FVS, FPS, SPS, TASS, and ORGANON twenty years ago and concluded that ORGANON "stood heads and shoulders above the rest." As a result, the company calibrated the ORGANON equations for a range of species and geographic areas, and the resulting simulator continues to be used to manage their extensive landholdings. Therefore, the ORGANON system of equations is still germane to the characterization of tree and forest development in today's world.

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### 1.0 General Description of the Data Sets to be Used to Illustrate the Development of the ORGANON Equations

Chapters 2 through 5 will use four data sets to explore the behavior of the four dynamic equations found in those chapters. These four data sets were used to develop two versions of ORGANON (SWO-ORGANON and SMC-ORGANON) at two different points in time over the course of each's data collection. They will be used to illustrate the effect that changes in population definition, sampling design, and measurement specifications can have upon the parameterization of diameter increment $(\Delta \mathrm{D})$, height increment $(\Delta \mathrm{H})$, probability of mortality (PM), and crown recession ( $\triangle \mathrm{HCB}$ ) equations.

Both editions of SWO-ORGANON used data collected on temporary research plots. The original version of SMC-ORGANON used data collected on permanent research installations, while the revised version of SMC ORGANON used data from both permanent research installations and, for modeling $\Delta \mathrm{D}$, the temporary research plots used for developing SWOORGANON. Pros and cons of the two approaches are:

1. Using temporary research plots, data on tree and stand development can be collected relatively quickly on a wide range of stand structures that could take decades to centuries to collect on permanent research plots.
2. Unless limited to a single growth period, determining tree and stand responses to treatments, such as thinning and fertilization, can only be done on permanent research installations that include untreated plots.
3. While it is possible to estimate when recent mortality occurred on temporary plots, mortality is more accurately measured on permanent plots.
4. Temporary research plots can be installed in operational stand conditions, while most permanent research plots are carefully created to minimize the variation found in operational stands. Ultimately, the user of a growth and yield model is interested in projecting tree and stand development in operational stands.

This chapter will end with the presentation of three alternative attempts that I made to replace SI as a measure of productivity in the ORGANON model.

### 1.1 SWO-ORGANON Data Set

### 1.1.1 The Original Data Set

The initial data for this study were collected during the summers of 1981, 1982, and 1983 as part of the southwest Oregon Forestry Intensified Research (FIR) Growth and Yield Project (Hann and Larsen 1991). The study area extended from near the California border ( $42^{\circ} 10^{\prime} \mathrm{N}$ ) on the south to the Cow Creek drainage ( $43^{\circ} 00^{\prime} \mathrm{N}$ ) on the north and from the Cascade crest ( $122^{\circ} 15^{\prime} \mathrm{W}$ ) on the east to approximately 15 miles west of Glendale (123'50'W).

Temporary plots were established within 391 stands selected from the study area. The following criteria were used to select these stands:

1. The majority of the trees in the stand must have an age under 120 years old when measured at breast height;
2. The majority of the trees in the stand must be either Douglas-fir, white fir, grand fir, ponderosa pine, sugar pine, incense cedar, or a mixture of them;
3. The stand must have a uniform stand structure so that the species mix, competitive structure, and resulting potential management practices are essentially unchanged throughout the stand;
4. The stand must have a common bedrock, landform, and soil series, and be similar in aspect, slope, and elevation throughout the stand;
5. The stand must not have been treated within the past 5 years.

Within each stand, a cluster of from 4 to 10 variable-radius plots and 2 nested, fixed-area subplots was installed in a random fashion to measure the attributes on all trees taller than 6 inches high. A variable-radius plot with a basal area factor of 20 was used for trees with an 8.1 -inch or greater value of diameter at breast height (D) at the end of the growth period $\left(D_{2}\right)$; a circular fixed-area subplot with a radius of 15.56 feet was used for trees with a 4.1- to 8 -inch $D_{2}$; and a circular fixed-area subplot with a radius of 7.78 feet was used for trees with a $D_{2}$ of 4 inches or less.

Tree measurements taken at the end of the most recent 5-year growth period (i.e., measurements subscripted with a 2) included a mortality indicator of whether the tree died in the past 5 years, $\mathrm{D}_{2}$, total tree height $\left(\mathrm{H}_{2}\right)$, height to live-crown base $\left(\mathrm{HCB}_{2}\right)$, and horizontal distance from plot center to tree center (DIST). In addition, past 5-year radial growth and $\Delta \mathrm{H}$ were measured on subsamples of the trees.

The dating of when trees died was based upon physical features of the dead tree as described in USDA Forest Service (1978) and Cline et al. (1980). $D_{2}$ was measured to the nearest 0.1 inch with a diameter tape. Both $\mathrm{H}_{2}$ and $\mathrm{HCB}_{2}$ were measured by the tangent method (Curtis and Bruce 1968, Larsen et al. 1987). The position of the base of the crown was determined by visual reconstruction of the crown such that any gaps in the crown were filled-in with branches from below the crown base. The distance from plot center to tree center was determined by measuring the horizontal distance from plot center to tree face and then adding one-half $\mathrm{D}_{2}$, expressed in feet, to it.

Past 5-year radial growth at breast height was measured with an increment borer on all trees with a $D_{2}$ large enough to accept the borer (approximately 2 inches). The boring occurred on the side of the tree facing plot center, and the resulting core was measured to the nearest 40th of an inch, ignoring the current year's growth. The inside-bark radial growth measurements were converted to outside-bark $\Delta \mathrm{D}$ by using the prediction equations for the ratio of diameter inside bark to diameter outside bark as developed for the six targeted conifer species of southwest Oregon by Larsen and Hann (1985) and for California hardwoods by Pillsbury and Kirkley (1984). Finally, the $\Delta \mathrm{D}$ measurements for the six targeted conifer species were adjusted, by using the equation presented in Zumrawi (1990), to eliminate the measurement
bias that occurs when increment borings are used instead of repeated measurements of $D$ to determine outside-bark $\Delta \mathrm{D}$.

All undamaged trees shorter or equal to 25 feet tall were measured for 5 -year $\Delta \mathrm{H}$ with a 25foot telescoping pole. For trees taller than 25 feet, a subsample of up to six trees on each plot was felled and sectioned at the first and sixth whorls; the ages at these whorls were determined to ensure a true 5-year growth period, and finally the distance between the two whorls was measured for 5-year $\Delta \mathrm{H}$.

Because the objective of the project is to predict future rather than past tree development, it was necessary to backdate all of the tree measurements in order to estimate their values at the start of the previous 5 -year growth period (i.e., $\mathrm{D}_{1}, \mathrm{H}_{1}$, and $\mathrm{HCB}_{1}$ ). The procedures used to backdate the tree measurements are described in detail in the Appendix 1.4.1. For trees that died during the growth period, it was assumed that the values at the start of the growth period were the same as those at the end of it.

Finally, site index ( $\mathrm{SI}_{\mathrm{HS}}$ ) for each plot was determined using the sampling procedures and equations found in Hann and Scrivani (1987).

Extensive effort was put into data quality procedures during the data collection process. This effort included:

1. The development of manuals detailing how the measurements should be conducted and it was expected that the crews would carry these manuals into the woods and refer to them when questions arose.
2. Using these manuals, extensive training sessions, both indoors and in the woods, were conducted with the measurement crews before the measurement season started in order to explain the correct measurement protocols that the crews were expected to follow.
3. Data collected by the crews were sent weekly to headquarters for computer conducted data editing in order to find data collection problems in a timely fashion.
4. The data edits were then shared with the crews by the crew managers and the plots were remeasured if needed.
5. The crew managers also remeasured a proportion of the plots previously measured by the crews and the results were also shared with the crews. Again, the plots were remeasured if needed.

The objective was to collect the highest quality modeling data possible. When the resulting data set was shared with one of Boise Cascade's biometricians, he informed me that the SWO data set was the best quality data set that he had ever worked with. This mission was truly accomplished!

### 1.1.2 The Revised Data Set

In response to the listing of the northern spotted owl (Strix occidentalis) as a threatened species under the Endangered Species Act of 1973, research was started in southwest Oregon 1) to identify stand structures and spatial relationships that are utilized effectively by the northern spotted owl and could contribute to the maintenance of a stable population over time, and 2) to develop silvicultural systems and associated mensurational tools needed to implement this knowledge at the stand level. One major mensurational tool needed to manage northern spotted owl habitat was the extension of SWO-ORGANON and its associated diameter-growth-rate equations to include stands with older trees (250 years or more), a higher component of hardwood species, and more complex spatial structure than were included in the original version.

The second study covered the same general area, but extended the selection criteria to include stands with trees >250 years old and to younger stands with a greater component of hardwoods. An additional 138 plots were measured in the second study. Sampling design and sample measurement protocols that changed in the second study included:

1. The addition of a 60-BAF variable radius subplot to select measurement trees in which $\mathrm{D}_{2}>36.0$ inches and, as a result, trees with $\mathrm{D}_{2}>36.0$ in the original data set were dropped if they did not fall on the 60-BAF plot;
2. The usage of a $45-\mathrm{ft}$ telescoping fiberglass pole instead of a 25 -foot plot to either directly measure $\mathrm{H}_{2}$ and $\Delta \mathrm{H}$ or to measure $\mathrm{H}_{2}$ indirectly using the tangent method used in the first study.
3. The elimination of sample plots from both the original data and the revised data set that were significantly impacted by past cutting.

Of the 529 plots in the combined data set, 526 plots contained at least one Douglas-fir that was above breast height. However, only 407 of the plots were not affected by cutting and contained Douglas-fir large enough to be increment bored. A description of the revised (and improved) backdating process used for the revised data set can be found in the Appendix 1.4.1.

The same data collection quality control procedures used in the original study were also applied in the collection of the revised data set.

### 1.2 SMC-ORGANON Data Set

### 1.2.1 The Original Data Set

The primary goal of the SMC Modeling Project was to develop H from D (i.e., H/D), HCB, $\triangle \mathrm{D}$, $\Delta \mathrm{H}, \mathrm{PM}$, and maximum size-density trajectory equations for Douglas-fir and western hemlock trees growing in southwestern British Columbia, western Washington, and northwestern Oregon. These equations were to include appropriate responses to fertilization and thinning and would form the basis for a new version of ORGANON (i.e., SMC-ORGANON). Where appropriate, the equations were also to include crown size to allow connections to the woodquality work of the SMC.

The project accumulated a database of 3,345 plots from 371 installations in the study area. Of these, 1,269 plots contained no western hemlock, 389 contained no Douglas-fir, and 1,687 contained both species. The installations ranged from $42.00^{\circ} \mathrm{N}$ to $50.63^{\circ} \mathrm{N}$ in latitude and from $120.7^{\circ} \mathrm{W}$ to $127.68^{\circ} \mathrm{W}$ in longitude. The data were collected from fixed-area plots averaging 0.17 acre and ranging in size from 0.1 to 1.2 acre. The average breast height age (BHA) was 27.8 year and ranged from 3 to 108 year. Various thinning and fertilization treatments were represented, although most were research, rather than operational, treatments. All plots were measured at least twice. Length of the growth periods between measurements ranged from 1 to 27 years, with an average of 4.5 years.

Attributes measured included diameter D for all sample trees and all measurement times; an indicator of whether the tree had died during the previous growth period for all trees alive at the start of the previous growth period; H for a subsample of the trees measured; and, on some installations, HCB a subsample of the trees measured (usually the same trees that were measured for h). D was measured to the nearest 0.1 inch ( 0.1 cm in British Columbia) with a diameter tape. H was measured by unknown techniques that could have varied from one installation to another. Unmeasured values of H were estimated with the plot-level heightdiameter fitting procedures of Flewelling and De Jong (1994), combining treatments within an installation whenever possible. H was measured on a subsample of trees; measurement techniques and definitions of the location of H are unknown. All data measured in metric units were stored in that format and converted to English units during the creation of the modeling data sets.

The ORGANON software projects stand development over a 5 -year growth period. The approach used to model $\Delta \mathrm{D}$ and $\Delta \mathrm{H}$ requires exactly 5 -year data, whereas the approach taken to model mortality rate allows the use of data with variable lengths of growth period. The following approach was used to calculate exact 5 -year growth periods for installations in which the total duration of measurements equaled or exceeded 5 years:

1. Starting with the first measurement, lengths of growth periods were cumulated until a total of 5 year was met or exceeded.
2. If a 5 -year growth period was exceeded by no more than 2 years, linear interpolation was applied to the measured changes in D and H during the last growth period in the accumulation. The appropriate fractional value of these measured changes was added
to the values at the start of the of the last growth period in order to calculate D and H at the end of a 5-year growth period. If, for example, D for a tree was measured every 2 years over 6 years, with resulting values of $6.0,6.6,7.1$, and $7.5 \mathrm{in} ., \mathrm{D}$ at the end of the 5 -year growth period would be calculated as 7.3 in. (i.e., $7.1+(7.5-7.1) / 2$ ) and the resulting 5 -year $\Delta \mathrm{D}$ growth rate would be 1.3 in . (i.e., $7.3-6.0$ ).
3. The process then proceeded to the next measurement (i.e., all 5-year measurement periods started with actual measurements, rather than interpolations), and steps 1 and 2 were repeated until either there were no additional growth periods available or the accumulation for the last period was less than 5 years. (4) For SMC installations in which the total duration of measurements was only 4 years, linear extrapolation was used to calculate D and H at the end of the 5 -year growth period by multiplying the measured 4 -year changes by 1.25 and adding these expanded increments to the D and H at the start of the growth period. Cumulated growth periods $<4$ years were discarded.

The resulting data sets gathered for Douglas-fir and western hemlock were evaluated for their adequacy in developing the $\Delta \mathrm{D}, \Delta \mathrm{H}, \mathrm{PM}$, and HCB equations in ORGANON. Of particular interest was the adequacy of trees with measurements of $C R$, an important variable in the core equations of ORGANON. This evaluation indicated that the number of tree observations with measurements of $\mathrm{H}, \Delta \mathrm{H}$, and $C R$ was small. If the measurements were taken on the plots, they were always subsamples of the trees found on the plot. Hs were not always measured on the same tree over time, and they were often concentrated in dominant trees on the plot. Measurements of CR were restricted to those trees with at least one measurement of H . Subsampling was particularly severe in the fertilization data sets from the Regional Forest Nutrition Research Project (RFNRP), in which H measurements were restricted to four dominant trees on each 0.1 -ac plot and there were no measurements of CR. No CRs were measured on the other large fertilization data set made available by the British Columbia Ministry of Forests. These data problems largely dictated the analytical approaches taken to develop the four equations.

There were also problems with data from SMC Type 1 installations. These installations were established in young plantations (most with breast height ages $<10$ years) or recently respaced natural stands of homogeneous stocking. In these stands, the $\Delta \mathrm{H}$ rates are much greater than expected from the dominant height growth equations of Bruce (1981) and Bonner et al. (1995). As a result, predicting SI from the Bruce (1981) and Bonner et al. (1995) equations resulted in greatly inflated values for the SMC Type 1 installations. Various attempts were tried unsuccessfully to derive a "fix" for this problem; therefore, it decided not to use the SMC Type 1 installations in the analyses.

Finally, there were occasional installations or plots encountered for which the documentation, measurement, or both, of initial conditions, past treatments, or measurement techniques was inadequate. For installations without information on the trees removed at the initial spacing treatments, the initial conditions were estimated using data from the control plots, where possible. In some cases, data from the early measurements of a plot were rejected because of the presence of a large number of unmeasured small trees (as evidenced by later ingrowth). Where the problems could not be alleviated, the data were eliminated from further analysis.

As with the SWO-ORGANON modeling data sets, computer editing was conducted on the SMC-ORGANON modeling data sets and the data was found to be larded with measurement errors. Correcting these errors took a substantial amount of time, manpower, and, therefore, money to correct. Most frustrating was the fact that the original sources of the data did not incorporate these costly corrections into their base data sets so the errors continue to today.

### 1.2.2 The Revised Data Set

Since the original SMC-ORGANON equations were created, (1) the subsequent 12 years had allowed for additional plot establishment, remeasurement, and growth, and (2) a new dominant-height-growth equation had been produced that predominantly utilized data from SMC installations (Flewelling et al. 2001). Comparison of the dominant height- growth equation of Flewelling et al. (2001) to that of Bruce (1981) showed close agreement for total ages >15 years. Bruce's $\mathrm{SI}\left(\mathrm{SI}_{\mathrm{B}}\right)$ (Bruce 1981) can therefore be estimated by predicting dominant height from the equation of Flewelling et al. (2001) at a BHA of 50 yr . As a result, it was felt that a reasonable estimate of $\mathrm{SI}_{\mathrm{B}}$ could be determined on the Type I and Type III installations. Given these developments, the SMC decided to reanalyze the $\Delta \mathrm{D}, \Delta \mathrm{H}$, and PM equations for Douglas-fir in order to better characterize these equations in young plantations. The resulting new equations were inserted into a revised version of SMC-ORGANON and tested against the original version.

This reanalysis utilized seven data sets. Four came from the SMC, and three from data collected in previous ORGANON modeling work.

### 1.2.2.1 SMC Cooperator Data

The first SMC data set selected for this analysis was part of the SMC cooperator data used to develop the original version of SMC-ORGANON. All of this data came from untreated permanent plots in even-aged Douglas-fir stands on public and private ownerships throughout southwestern British Columbia, western Washington, and northwestern Oregon. The 19 installations containing these plots were originally established in both plantations and natural stands to explore a variety of silvicultural objectives. Plot sizes ranged from 0.05 to 1.0 acre, with the 0.2 -acre plot being most common. These $\Delta \mathrm{D}, \Delta \mathrm{H}$, and PM data sets are described fully by Hann et al. (2003).

### 1.2.2.2 SMC Installation Data

The Type I, II, and III installations of the SMC that had been established in pure Douglas-fir plantations were also used in this analysis. Total age (TA) at establishment ranged from 6 to 18 years on the 29 Type I installations, from 18 to 40 years on the 12 Type II installations, and from 5 to 10 years on the 21 Type III installations. The Type I and II installations each contained a single control plot of 0.5 ac . The Type III installations contained one control plot in each of the six planting densities (100, 200, 300, 440, 680, and 1,210 TPA) on an installation. Plot sizes on the Type III installations ranged from 0.496 ac for the 100-tpa planting density to 0.212 acre for the 1,210-TPA density. For all three types of SMC installations, the remeasurement intervals were either 2 or 4 years, and the total length of measurements ranged from 8 to 12 years. H and HCB were subsampled on all of the SMC installations. These $\Delta \mathrm{D}, \Delta \mathrm{H}$, and PM data sets are described fully by Hann et al. (2006).

The ORGANON software uses a 5-year growth period. The procedure used to model PM can directly use the 4-year measurement data to estimate PM (Hann et al. 2003). This is not true for estimating the $\Delta \mathrm{D}$ and $\Delta \mathrm{H}$ equations. Therefore, the interpolation and extrapolation procedures described in Section 1.4.2.1 were also used in the creation of the revised data set.

The SMC was one of the organizations that did not incorporate the data corrections from the original analysis into their data base. Therefore, only the data collected since the original analysis were gathered from the SMC and then the data were subjected to the computerized data checks and, usually, manual corrections of the data. The resulting corrected data was then added to the corrected SMC data from the original analysis.

### 1.2.2.3 ORGANON Data

The $\Delta \mathrm{D}$ analysis of Hann and Hanus (2002) showed that the model's predictive behavior could be substantially improved by including larger diameter trees in the analysis. Because the SMC data sets did not contain very large trees, it was decided to conduct a giant size regression analysis (Cunia 1973) by using indicator variables and including the data from three ORGANON modeling projects (southwest Oregon, northwest Oregon, and western Washington) in the development of the new SMC $\triangle \mathrm{D}$ equation. An added benefit from this giant size regression analysis is the creation of new $\Delta \mathrm{D}$ equations for the southwest Oregon and northwest Oregon versions of ORGANON.

The southwest Oregon data set is fully described in Section 1.1.2. The northwest Oregon study sampled 136 plots on the College of Forestry's McDonald-Dunn Research Forest using the same sampling procedures described in Section 1.1.1, with the exception that up to 25 sample points could be placed in the stand (Zumrawi and Hann 1993). Plots were predominantly evenaged in structure with at least $80 \%$ of their basal area in Douglas-fir. The western Washington study sampled 34 plots using the same sampling procedures described in Section 1.1.1, again with the exception that up to 25 sample points could be placed in the stand (McKenzie 1994). Plots were predominantly two-tiered in structure and composed primarily of Douglas-fir and western hemlock.

As with the southwest Oregon data set, measurements of $\mathrm{D}_{2}, \mathrm{H}_{2}$, and $\mathrm{HCB}_{2}$ were taken on all sample trees in the northwest Oregon and western Washington data sets. Backdating procedures for calculating $\mathrm{D}_{1}, \mathrm{H}_{1}, \mathrm{HCB}_{1}$ and $E F_{1}$ at the start of the growth period are described the Appendix for the northwest Oregon data set and in McKenzie (1994) for the western Washington data set. $\mathrm{SI}_{\mathrm{B}}$ was used in the northwest Oregon and western Washington data sets.

### 1.3 Attempts Made at Replacing Site Index as a Measure of Productivity in ORGANON

I believe that climate change will be a predominant driver of forest management decisions in the future. This will mean that the usage of SI, a phytocentric measure of productivity, will become less useful in the future as the relationship between past productivity, as indicated by SI, and future productivity become disconnected. Over my 40-year career, I have made three attempts to disconnect ORGANON from SI and connect it to attributes more closely related to the environment in which the trees grew.

### 1.3.1 First Attempt

My first attempt occurred in the mid 1980's during the development of the original edition of SWO-ORGANON. Because of the complex types of bedrock, soils, climate, and stand structures found in SWO, I decided to measure and compute 27 attributes and calculated variables related to productivity that are listed in Appendix 1.4.2. At the suggestion of the late Dr. Robert O. Curtis of the USFS, one of the reviewers of my study plan, I was persuaded to also collect stem analysis data on both Douglas-fir and ponderosa pine trees that were later used to develop the SI equations of Hann and Scrivani (1987).

The alternative productivity variables were evaluated for their utility in predicted diameter increment by using a number of all-combination, linear regression screening runs using the logarithm of diameter growth as the response variable. Various transformations of the alternative productivity measurements and variables were tried, with one set of runs including $\log$ of SI as a predictor variable and a separate set of runs without the usage of the log of SI . From these runs, I found that SI was the strongest productivity variable and that, while some alternatives were significant, no combination of them explained more than 3 or 4 percent of the variation when used alone or more than 2 percent when used with SI. Because of the considerable cost associated with collecting and computing many of the statistically significant alternative productivity variables, I decided not to include any of them in the final diameter growth equation. These results are presented in Hann and Larsen (1991).

### 1.3.2 Second Attempt

In my second attempt at incorporating an alternative productivity variable into ORGANON, I worked with Dick Waring and Nick Coops on a NASA project in the late 1990's to see if predictions of net photosynthesis from 3-PG could be used as a substitute for SI in SWOORGANON. I provided them with the latitudes and longitudes of the evenaged Douglas-fir stands from the SWO-ORGANON data set and they in turn provided me with predictions of net photosynthesis for each site. I then used regression analysis to determine if their estimate of net photosynthesis was related to each plot's measured five-year volume increment or to its SI value. All of the alternative equations explored in the analysis explained less than $10 \%$ of the variation in either volume increment or SI. I therefore concluded that 3-PG was not an adequate replacement of SI in SWO-ORGANON. Because these results were negative (i.e., application of 3-PG did not work), they were never published.

### 1.3.3 Third Attempt

In my third attempt at incorporating an alternative productivity variable into ORGANON occurred in 2011, I used the data from 149 Douglas-fir dominated stands in southwest Oregon to explore a number of site productivity descriptors for their utility in predicting diameter increment. The southwest Oregon data set was chosen because it had been collected in operational stands, using operational data collection procedures, and it covered a wider range of environments (Franklin and Dyrness 1973) than most research studies. The site productivity descriptors included three physiographic descriptors, seven soil and bedrock descriptors, 14 climate descriptors, and three solar radiation descriptors. Definitions of these variables are found in Appendix 1.4.3.

The latitude and longitude of each stand was used to determine the elevation at the center of the stand's sampling grid using USGS digital elevation models. The latitude and longitude values were also used to download climate data for each stand using DAYMET. These values were then used to calculate the average values of the relevant variables for each of the three 5 -year growth periods measured in the original study.

Two sets of equations were used to calculate total direct beam irradiance above the atmosphere. The equation of McCune and Keon (2002) uses aspect, slope, and latitude to estimate annual direct irradiance. The equations of Coops et al. (2000) also include the elevation and average maximum monthly temperature for each month to estimate annual direct irradiance for any growth period desired. These equations were used to estimate both the annual direct irradiance and the growing season direct irradiance for each plot.

After extensive screening, the two variables that were found to be both significantly different from zero at $\alpha=0.05$ and had biological reasonable parameter estimates were available waterholding capacity in top 20" of soil (WHC20) and growing season precipitation over degree days above $5^{\circ} \mathrm{C}$ (PPTDD5). The level of significance on the parameter for WHC20 is not a surprise given the droughty summers experienced in the Pacific Northwest (and particularly in southwest Oregon) which places a premium on the storage of water in the soil. PPTDD5
integrates the effects of both air temperature, through the definition of DD5, and amount of precipitation over that period.

When WHC20 and PPTDD5 were inserted as replacements for SI into both a diameter increment ( $\Delta \mathrm{D}$ ) equation and a height increment $(\Delta \mathrm{H})$ equation, they predicted 2.93 -percent more of the variation in $\Delta \mathrm{D}$ and 3.52-percent more of the variation in $\Delta \mathrm{H}$ than the model without WHC20 and PPTDD5. This compared with the value of 6.31 -percent for $\Delta \mathrm{D}$ and 9.84 percent for $\Delta \mathrm{H}$ when SI was used instead of WHC20 and PPTDD5. Therefore, SI explained more than double of the variation in $\Delta \mathrm{D}$ and almost three times more of the variation in $\Delta \mathrm{H}$ than WHC20 and PPTDD5. This finding is not surprising given that SI is a phytocentric measure of productivity and it helps to explain the duration of SI as a measure of productivity.

Of the two alternative productivity variables, WHC20 is the most problematic to use in application because of the need to dig 20 " soil pits in a stand in order to apply the equation (though digging to 20 " is substantially easier than the 54 " or bedrock used in the original SWO data collection effort). An attempt was made to substitute WHC20 estimated from digitized SCS soil maps but it was unsuccessful. On the positive side, the results of this third attempt show that real progress has been made over my 40 -year career in the ability to obtain climate data from programs such as DAYMET that are useful across stands growing on a complex set of environments located within a relatively small area such as SWO. Whether similar progress can be made with site specific soils data is yet to be determined, but one problem with using soil data is the large degree of variability in soil attributes within a stand.

I have long believed that if a successful biologically meaningful method for characterizing productivity in the complex environments of southwest Oregon could be found, then the method would be equally useful in less complex environments. However, it should be said that, while a number of alternative productivity related variables did not prove useful for characterizing the productivity of the complex environments in southwest Oregon, it does not follow that they would not be useful in less complex environments.

### 1.4 Appendices

### 1.4.1 Appendix: Backdating Procedures for Temporary Growth Plots in Western Oregon

The following are descriptions of the procedures used to backdate $D_{1}, H_{1}, H C B_{1}$, and $E F_{1}$ in two versions of ORGANON and two editions of one of the versions, which were developed using data from temporary plots. The descriptions are presented in chronological order so that the evolution in the procedures can be examined.

### 1.4.1.1 Backdating the NWO-ORGANON Data Set

The following procedures from Ritchie and Hann (1985) were applied to the tree measurements from each stand to backdate them to the start of the previous 5-year growth period. These backdating procedures were the first developed and used in the development of a version of ORGANON (ultimately NWO-ORGANON).

### 1.4.1.1.1 Determining $D_{1}$

$D_{1}$ was estimated from the equation:

$$
\begin{equation*}
D_{1}=\left[\frac{b_{1} D_{2}^{b_{2}}-2 \times R G}{b_{1}}\right]^{\frac{1.0}{b_{2}}} \tag{A-1}
\end{equation*}
$$

Where,
$R G=$ measured 5-year radial growth of the tree, inside bark,
$b_{1}$ and $b_{2}=$ regression coefficients from Larsen and Hann (1985) for predicting D inside bark from D outside bark.

### 1.4.1.1.2 Determining $\mathrm{H}_{1}$

For trees with a measured 5-year $\Delta \mathrm{H}, \mathrm{H}_{1}$ was determined by:
$\mathrm{H}_{1}=\mathrm{H}_{2}-\Delta \mathrm{H}$
For trees without a measured 5 -year $\Delta \mathrm{H}, \mathrm{H}_{1}$ was determined by:
$\mathrm{H}_{1}=\mathrm{H}_{2}-\mathrm{P} \Delta \mathrm{H}_{\mathrm{B}}$
Where,
$P \Delta H_{B}=\mathrm{H}_{2}-\mathrm{f}_{\mathrm{B}}\left[\mathrm{SI}_{\mathrm{B}}, G E A-5.0\right]$
$\mathrm{GEA}=\mathrm{f}_{\mathrm{B}}{ }^{-1}\left[\mathrm{SI}_{\mathrm{B}}, \mathrm{H}_{2}\right]$
$f_{B}=$ Bruce's (1981) dominant height growth equation

### 1.4.1.1.3 Determining $\mathrm{HCB}_{1}$

$\mathrm{HCB}_{1}$ was computed with the following equation:
$\mathrm{HCB}_{1}=\left(\mathrm{H}_{1} / \mathrm{H}_{2}\right) \times \mathrm{HCB}_{2}$
This equation is based on the assumption that crown ratio (i.e., $1.0-\mathrm{HCB} / \mathrm{H}$ ) is constant for short growth periods.

### 1.4.1.1.4 Determining $\mathrm{EF}_{1}$

The expansion factor (number of trees per acre represented by a sampled tree) at the start of the growth period $\left(E F_{1}\right)$ was based on $\mathrm{D}_{1}$, distance to the center of the tree (DIST), and the following rules derived from the sampling design:

1. If $\mathrm{D}_{1} \leq 4.0$ inches and DIST $<7.78$ feet, $\mathrm{EF}_{1}=229.18$ trees per acre: otherwise $\mathrm{EF}_{1}=$ 0.0 .
2. If $\mathrm{D}_{1}>4.0$ inches but $\leq 8.0$ inches and DIST $<15.56$ feet, $\mathrm{EF}_{1}=57.30$ trees per acre; otherwise $\mathrm{EF}_{1}=0.0$.
3. If $\mathrm{D}_{1}>8.0$ inches, a critical distance (CDIST) is first computed by CDIST $=1.944544$ $\left(\mathrm{D}_{1}\right)$. If DIST $\leq$ CDIST, $E F_{1}$ is computed by $E F_{1}=3666.93\left(\mathrm{D}_{1}\right)^{-2}$; otherwise $\mathrm{EF}_{1}=0.0$.

### 1.4.1.2 Backdating the Original SWO-ORGANON Data Set

The following procedures from Hann and Wang (1990) were applied to the tree measurements from each stand to backdate them to the start of the previous 5 -year growth period. These backdating procedures were the second developed and used in the development of a version of ORGANON (the original SWO-ORGANON).

### 1.4.1.2.1 Determining $D_{1}$

If a measurement of the 5 -year radial growth inside bark (RG) of the tree was available, then $D_{1}$ was estimated from Equation ( $\mathrm{A}-1$ ).

Furthermore, the data for each tree species in which RG was measured on at least five trees on each plot were used to develop the following species specific regression equation:
$\ln \left(\right.$ TBAG $\left._{i b}\right)=a_{0,1}+a_{1,1} D_{2}^{2}+a_{1,2} \ln \left(D_{2}\right)$
where,
$\mathrm{TBAG}_{i b}=$ basal area growth inside bark
$\mathrm{TBAG}_{i b}=\pi R G \times\left(\mathrm{b}_{1} \mathrm{D}_{2}^{\mathrm{b} 2}-R G\right)$
$\mathrm{a}_{0,1}$ and $\mathrm{a}_{1,1}=$ regression coefficients determined from the data
Equation (A-3) is a simplified version of a form that has been successfully used by Wykoff et al. (1982), Ritchie and Hann (1985), Johnson et al. (1986) and Wykoff (1986) to predict basal area growth of individual trees. For species in which RG had been measured in fewer than five trees, species with similar growth forms were combined until at least five observations were available to estimate the parameters.

Equation (A-3) was used to estimate $D_{1}$ for trees In the stand without a measured $R G$ by applying the following relationship:

$$
D_{1}=\left[D_{2}^{2 b_{2}}-\frac{4.0 \times T B A G_{i b}}{\pi b_{1}{ }^{2}}\right]^{\frac{1.0}{2 \times b_{2}}}
$$

### 1.4.1.2.2 Determining $\mathrm{H}_{1}$

For trees with a measured 5-year $\Delta H, H_{1}$ was determined by Equation (A-2).
For each stand, one or two equations were developed for each species to predict $\mathrm{H}_{1}$ for trees without a measured 5 -year $\Delta \mathrm{H}$. The number and type of equations developed for a particular species depended on the number of trees on the plot with $\mathrm{H}_{2} \geq 20$ feet, the number of trees with $\mathrm{H}_{2}<20$ feet, and the number of trees $<20$ feet tall with measured 5 -year $\Delta \mathrm{H}$. Two sets of equations were used, depending upon $\mathrm{H}_{2}$, for two reasons:

1. It was felt that developing two sets of equations would increase the precision of backdating the heights of shorter trees.
2. $\Delta \mathrm{H}$ of trees $<25$ feet tall were measured directly with the telescoping pole: as a result, more $\Delta \mathrm{H}$ measurements were taken of the shorter trees. This opened the possibility of developing a stand-specific $\Delta H$ equation.

If six or more trees of a given species in a stand each had a $\mathrm{H}_{2} \geq 20$ feet, the following heightdiameter equation was fit to the data by least squares regression:
$\ln \left(\mathrm{H}_{2}-4.5\right)=\mathrm{a}_{0,2}+\mathrm{a}_{1,2} \mathrm{D}_{2}{ }^{-1}$
This equation is very similar to that used by Wykoff et al. (1982). $\mathrm{H}_{1}$ was estimated for all trees of the same species without measured 5 -year $\Delta H$ that were $\geq 20$ feet high by Equation (A-4) and the relationship:
$\mathrm{H}_{1}=\mathrm{H}_{2} \times\left(\mathrm{PH}_{1} / \mathrm{PH}_{2}\right)$
Where,

$$
\begin{aligned}
& \mathrm{PH}_{1}=4.5+\operatorname{EXP}\left(\mathrm{a}_{0,2}+\mathrm{a}_{1,2} \mathrm{D}_{1}^{-1}\right) \\
& \mathrm{PH}_{1}=4.5+\operatorname{EXP}\left(\mathrm{a}_{0,2}+\mathrm{a}_{1,2} \mathrm{D}_{2}^{-1}\right)
\end{aligned}
$$

If a measured 5 -year $\Delta H$ was available for at least six trees of a given species $<20$ feet tall, the following height-growth equation was developed by least squares regression:
$\ln (\Delta H)=\mathrm{a}_{0,3}+\mathrm{a}_{1,3} \ln \left(\mathrm{H}_{2}\right)$
Wykoff et al. (1982) used Equation (A-5) to characterize height-growth rates of trees with DBH $<3$ inches.

For trees< 20 feet tall without a 5 -year $\Delta H$ measurement, $H_{1}$ was estimated by
$\mathrm{H}_{1}=\mathrm{H}_{2}-\operatorname{EXP}\left[\left(\mathrm{a}_{0,3}+\mathrm{a}_{1,3} \ln \left(\mathrm{H}_{2}\right)\right]\right.$
If measured 5 -year $\Delta H$ s were not available for at least six trees $<20$ feet tall, but at least six trees of the given species were $<20$ feet tall, the following allometric equation was fit to the data by least squares regression:
$\ln \left(\mathrm{H}_{2}-4.5\right)=\mathrm{a}_{0,4}+\mathrm{a}_{1,4} \ln \left(\mathrm{D}_{2}\right)$
$\mathrm{H}_{1}$ was estimated for all trees of the same species that were $<20$ feet tall and on which 5 -year $\Delta \mathrm{H}$ had not been measured by using Equation (A-7) and the relationship:

$$
\begin{equation*}
\mathrm{H}_{1}=\mathrm{H}_{2 \times} \times\left(\mathrm{PH}_{1} / \mathrm{PH}_{2}\right) \tag{A-8}
\end{equation*}
$$

Where,

$$
\begin{aligned}
& \operatorname{PH} 2_{1}=4.5+\operatorname{EXP}\left[\left(\mathrm{a}_{0,4}+\mathrm{a}_{1,4} \ln \left(\mathrm{D}_{1}\right)\right]\right. \\
& \operatorname{PH} 2_{2}=4.5+\operatorname{EXP}\left[\left(\mathrm{a}_{0,4}+\mathrm{a}_{1,4} \ln \left(\mathrm{D}_{2}\right)\right]\right.
\end{aligned}
$$

If fewer than six trees in a species had $\mathrm{H}_{2} \geq 20$ feet, the height distinction was eliminated and all trees were combined. If the combined data for a species provided at least six measured 5year $\Delta H$, Equation (A-5) was fit to the data and Equation (A-6) was used to estimate $H_{1}$ for trees without a measured 5 -year $\Delta H$. If the combined data for a species did not have at least six measurements of 5 -year height growth but did have at least six trees, Equation (A-7) was fit to the data and Equation (A-8) was used to estimate $\mathrm{H}_{1}$ for trees without a measured 5 -year $\Delta \mathrm{H}$. Finally, if a species had fewer than six trees, the data for the species were combined with data from similar species and appropriate regression equations were developed.

### 1.4.1.2.3 Determining $\mathrm{HCB}_{1}$

$\mathrm{HCB}_{1}$ was computed with the following equation:
$\mathrm{HCB}_{1}=\mathrm{H}_{1}-\left(\mathrm{H}_{2}-\mathrm{HCB}_{2}\right)$

This equation is based on the assumption that crown length (i.e., $\mathrm{H}-\mathrm{HCB}$ ) is constant for short growth periods.

### 1.4.1.2.4 Determining $E F_{1}$

The expansion factor (number of trees per acre represented by a sampled tree) at the start of the growth period $\left(E F_{1}\right)$ was based on $D_{1}$, distance to the center of the tree (DIST), and the following rules derived from the sampling design:

1. If $\mathrm{D}_{1} \leq 4.0$ inches and DIST $<7.78$ feet, $\mathrm{EF}_{1}=229.18$ trees per acre: otherwise, $\mathrm{EF}_{1}=$ 0.0 .
2. If $\mathrm{D}_{1}>4.0$ inches but $\leq 8.0$ inches and DIST $<15.56$ feet, $\mathrm{EF}_{1}=57.30$ trees per acre; otherwise, $\mathrm{EF}_{1}=0.0$.
3. If $D_{1}>8.0$ inches, a critical distance (CDIST) is first computed by CDIST $=1.944544$ $\left(D_{1}\right)$. If DIST $\leq$ CDIST, $E F_{1}$ is computed by $E F_{1}=3666.93\left(D_{1}\right)^{-2}$; otherwise, $E F_{1}=0.0$.

### 1.4.1.3 Backdating The Revised SWO-ORGANON Data Set

The following procedures from Hann and Hanus (2001) were applied to the tree measurements from each stand to backdate them to the start of the previous 5-year growth period. These backdating procedures were the third developed and used in the development of a version of ORGANON (the revised SWO-ORGANON).

### 1.4.1.3.1 Determining $D_{1}$

As with the original SWO-ORGANON data set, $D_{1}$ was estimated from Equation (A-1) when RG was measured on the tree.

Two alternative methods were used to compute $D_{1}$ when radial growth was not measured on the tree. If a $5-y r \Delta H$ measurement was available for the tree, then $D_{1}$ was computed as:
$\mathrm{D}_{1}=\mathrm{D}_{2} \times\left(\mathrm{H}_{2}-\Delta \mathrm{H}-4.5\right) /\left(\mathrm{H}_{2}-4.5\right)$
This relationship assumed that the D-to-H ratio remained constant over the 5-yr growth period.
If $\Delta H$ was not measured, a $\Delta \mathrm{D}$ calibration value ( $\triangle \mathrm{DCAL}$ ) for the predicted future $5-\mathrm{yr} \Delta \mathrm{D}$ (PAD) from the equations of Hann and Larsen (1991) was calculated for each species in the stand that had at least five values of measured $\Delta \mathrm{D}(\mathrm{M} \Delta \mathrm{D})$, as follows:
$\Delta D C A L=\sum M \Delta D_{i} / \sum P \Delta D_{i}$
$\triangle D C A L$ is the slope parameter in the simple linear equation $M \Delta D=\triangle D C A L \times P \Delta D$ that has been estimated using weighted regression and a weight of $\mathrm{P} \Delta \mathrm{D}^{-1}$ (Draper and Smith 1998).

For species in which $\Delta \mathrm{D}$ had been measured on less than five trees in the stand, an average calibration factor across all species was used. $\mathrm{D}_{1}$ was then computed for trees without either a measured RD or $\Delta \mathrm{H}$ by:
$D_{1}=D_{2}-\Delta D C A L \times P \Delta D$

### 1.4.1.3.2 Determining $\mathrm{H}_{1}$

For trees with a measured 5 -year $\Delta H, H_{1}$ was determined by Equation (A-2).
The approach chosen for predicting $\mathrm{H}_{1}$ for a tree without a measured 5 -yr $\Delta \mathrm{H}$ depended upon species, the severity of any damage to the tree, and whether the tree had a missing or dead top. Two initial estimates of $\Delta H$ were made for Douglas-fir, grand/white firs, ponderosa pine, sugar pine, and incense-cedar. The first estimate ( $\mathrm{E} 1 \Delta \mathrm{H}$ ) used the $\Delta \mathrm{H}$ equations of Ritchie and Hann (1990) to estimate the future $5-\mathrm{yr} \Delta \mathrm{H}$ at the end of the growth period. A calibration value ( $\Delta \mathrm{HCAL}$ ) for the Ritchie and Hann (1990) equation was also calculated for each species in the stand having at least five values of measured $\Delta H(M \Delta H)$ :
$\Delta H C A L=\Sigma M \Delta H_{i} / \sum E 1 \Delta H_{i}$
$\Delta H C A L$ is the slope parameter in the simple linear equation $M \Delta H=\Delta H C A L \times E 1 \Delta H$ that has been estimated using weighted regression and a weight of $\mathrm{P} \Delta \mathrm{H}^{-1}$ (Draper and Smith 1998).

For species in which height growth rate had been measured on less than five trees in the stand, an average $\triangle H C A L$ across all species was used.

The second initial estimate was used for all species, and applied the height/diameter equations of Larsen and Hann (1987) as follows:
$\mathrm{E} 2 \Delta \mathrm{H}=\mathrm{H}_{2}-\left[4.5+\left(\mathrm{PH}_{1}-4.5\right) /\left(\mathrm{PH}_{2}-4.5\right) \times\left(\mathrm{H}_{2}-4.5\right)\right]$
where
$\mathrm{PH}_{1}=$ Predicted $\mathrm{H}_{1}$ from $\mathrm{D}_{1}$ for a particular species, using the height/diameter equations of Larsen and Hann (1987)
$\mathrm{PH}_{2}=$ Predicted $\mathrm{H}_{2}$ from $\mathrm{D}_{2}$ for a particular species, using the height/diameter equations of Larsen and Hann (1987)

For Douglas-fir, grand/white firs, ponderosa pine, sugar pine, and incense-cedar, the predicted $\Delta H(P \Delta H)$ was set to $E 1 \Delta H$ if the tree was undamaged; to the average of $E 1 \Delta H$ and $E 2 \Delta H$ for trees with light damage; and to E2 $\Delta \mathrm{H}$ for severely damaged trees. For all other species, $\mathrm{P} \Delta \mathrm{H}$ was set equal to $\mathrm{E} 2 \Delta \mathrm{H}$ and $\Delta \mathrm{HCAL}$ was set equal to 1.0 . For all species, $\mathrm{P} \Delta \mathrm{H}$ was set to 0.0 for trees with missing or dead tops. With $\mathrm{P} \Delta \mathrm{H}$ set, $\mathrm{H}_{1}$ was then estimated by:

$$
\mathrm{H}_{1}=\mathrm{H}_{2}-(\Delta \mathrm{HCAL}) \times(\mathrm{P} \Delta \mathrm{H})
$$

### 1.4.1.3.3 Determining $\mathrm{HCB}_{1}$

$\mathrm{HCB}_{1}$ was computed via the HCB equations of Ritchie and Hann (1987). First, the predicted crown ratios at both the start $\left(\mathrm{PCR}_{1}\right)$ and the end $\left(\mathrm{PCR}_{2}\right)$ of the growth period were computed from the following relationship to HCB:
$P C R=1.0-\mathrm{PHCB} / \mathrm{PH}$
$\mathrm{PCR}_{2}$ was then calibrated to each species in the stand having a measured $\mathrm{CR}_{2}$, using the following equation:

CRCAL $=\left[\Sigma\left(\mathrm{PCR}_{2}\right)\left(\mathrm{MCR}_{2}\right)\right] /\left[\left(\mathrm{PCR}_{2}\right)^{2}\right]$
where
CRCAL = Calibration factor for a particular species in the stand having at least five values of $\mathrm{MCR}_{2}$
$\mathrm{MCR}_{2}=$ Measured CR at the end of the growth period for a particular species
CRCAL is the slope parameter in the simple linear equation $\mathrm{MCR}_{2}=\mathrm{CRCAL} \times \mathrm{PCR}_{2}$ that has been estimated using unweighted regression (Draper and Smith 1998).

The predicted change in HCB ( $\mathrm{P} \triangle \mathrm{HCB}$ ) could then be computed by:
$\mathrm{P} \Delta \mathrm{HCB}=\mathrm{H}_{2}\left[1.0-\left(\mathrm{PCR}_{2}\right)(\mathrm{CRCAL})\right]-\mathrm{H}_{1}\left[1.0-\left(\mathrm{PCR}_{1}\right)(\mathrm{CRCAL})\right]$
Finally, $\mathrm{HCB}_{1}$ was computed by:
$\mathrm{HCB}_{1}=\mathrm{HCB}_{2}+\mathrm{P} \Delta \mathrm{HCB}$
If $\mathrm{HCB}_{1}$ was predicted to be greater than $0.95 \mathrm{H}_{1}$, then $\mathrm{HCB}_{1}$ was set equal to $0.95 \mathrm{H}_{1}$.

### 1.4.1.3.4 Determining $E F_{1}$

$E F_{1}$ was used to calculate a number of point-level and stand-level attributes at the start of the growth period. The value of $\mathrm{EF}_{1}$ was based on $\mathrm{D}_{1}$, DIST, and the following rules derived from the sampling design:

1. For a tree with $D_{1} \leq 4.0$ inches, if DIST is $\leq 7.78$ feet, then $E F_{1}$ is 229.18 TPA; otherwise $E F_{1}$ is 0.0 .
2. For a tree with $D_{1}>4.0$ inches but $\leq 8.0$ inches, if DIST is $\leq 15.56$ feet, then $E F_{1}$ is 57.30 TPA; otherwise $E F_{1}$ is 0.0 .
3. For a tree with $\mathrm{D}_{1}>8.0$ inches and $\leq 36$ inches, a critical distance (CDIST20) is first computed by CDIST20 $=1.944544 \times \mathrm{D}_{1}$. If the tree's DIST is $\leq$ CDIST20, then $\mathrm{EF}_{1}=$ $3666.63\left(D_{1}\right)^{2}$; otherwise, $E F_{1}$ is 0.0 .
4. For a tree with $\mathrm{D}_{1}>36.0$ in., a critical distance (CDIST60) is first computed by CDIST60 $=1.122683 \times D_{1}$. If the tree's DIST is $\leq C D I S T 60$, then $E F_{1}=11000.79\left(D_{1}\right)^{2}$; otherwise $E F_{1}$ is 0.0 .

If $E F_{1}$ was zero, the tree was excluded from the analysis.

### 1.4.2 Appendix: Productivity Measurements Used in The First Attempt to Replace Site Index

Basic productivity related measurements collected in Southwest Oregon:

1. Stand elevation
2. Bedrock type from maps
3. Latitude of the plot center taken to the nearest second
4. Longitude of the plot center taken to the nearest second
5. Annual precipitation
6. Growing season prescription
7. Aspect of each sample point on the plot
8. Slope of each sample point on the plot
9. Depth of each horizon found in a modal soil pit dug to 54 " or bedrock
10. Texture for each horizon found in a modal soil pit dug to 54 " or bedrock
$11 . \%$ coarse fragments $>3$ " for each horizon found in a modal soil pit dug to 54 " or bedrock
$12 . \%$ coarse fragments between 2 mm to $3^{\prime \prime}$ for each horizon found in a modal soil pit dug to 54 " or bedrock
11. Rooting depth found in a modal soil pit dug to 54 " or bedrock
12. bedrock type found in a modal soil pit dug to 54 " or bedrock
$15 . \%$ slope to blocking ridge at azimuth of $90^{\circ}$
$16 . \%$ slope to blocking ridge at azimuth of $150^{\circ}$
$17 . \%$ slope to blocking ridge at azimuth of $210^{\circ}$
$18 . \%$ slope to blocking ridge at azimuth of $270^{\circ}$
The basic soils measurements were then used to compute the following variables:
13. Total waterholding capacity of soil
14. Available waterholding capacity in top 20 of soil
15. \% total coarse fragments in soil
16. \% total coarse fragments in top 20 " of soil
17. Total depth of soil
18. Effective depth of soil

Finally, the above measurements and variables were then used to compute other productivity variables at each site such as:

1. Average monthly minimum and maximum temperatures
2. Amount of solar irradiation received (Kaufmann and Weatherred 1982)
3. Amount of net photosynthesis produced (Emmingham and Waring 1977)
1.4.3 Appendix: Productivity Variables Used in the Third Attempt to Replace Site index

| Variable | Description |
| :---: | :---: |
| Physiographic Descriptors |  |
| SL | Slope |
| ASP | Aspect from north |
| EL | Elevation |
| Soil Descriptors from Modal Soil Pits |  |
| WHC20 | Available water holding capacity in top 20" of soil. |
| TWHC | Total available water holding capacity of soil |
| ED | Effective depth of soil |
| TD | Total depth of soil |
| TCF | \% total coarse fragments in soil > 2 millimeters |
| TCF20 | \% total coarse fragments in soil > 2 millimeters in the top 20" of Soil |
| BRI | Bed rock indicator variables |
| DATMET Descriptors |  |
| DA5 | Number of days above $5^{\circ} \mathrm{C}$ |
| NFFD | Number of frost free days |
| FFD | Frost free period |
| DD5 | Growing degree days with base temperature of $5^{\circ} \mathrm{C}$ |
| DD10 | Growing degree days with base temperature of $10^{\circ} \mathrm{C}$ |
| PPTFFP | Growing season precipitation over frost free period |
| PPTDD5 | Growing season precipitation over degree days above $5^{\circ} \mathrm{C}$ |
| MSP | Mean annual summer precipitation, 5/1 Through 9/30 |
| MSPA | Mean annual summer precipitation, 5/1 Through 8/15 |
| MSPB | Mean annual summer precipitation, 5/1 Through 9/15 |
| AHM | Average annual heat to moisture index |
| SHM | Average summer heat to moisture index |
| EREF | Average Hargreaves reference evaporation over growth period |
| CMD | Average Hargreaves climatic moisture deficit over growth period |
| Total Direct Beam Irradiance Above the Atmosphere Descriptors |  |
| Y_TRAD1 | Total direct irradiance above the atmosphere in kilo-calories $/ \mathrm{cm}^{2}$ from the yearly equation of McCune and Keon (2002) over the entire year |
| Y_TRAD2 | Total direct irradiance above the atmosphere in kilo-calories/ $\mathrm{cm}^{2}$ from the daily equation of Coops et al. (2000) over the entire year |
| P_TRAD | Total direct irradiance above the atmosphere in kilo-calories/cm ${ }^{2}$ from the daily equation of Coops et al. (2000) over the growth interval defined by DA5 |

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### 2.0 Diameter Increment Equation ( $\Delta \mathrm{D}$ )

For plots in which trees have grown above breast height, it is very common for diameter at breast height ( D ) to be measured on all trees because it is easier to measure than other tree attributes of interest such as tree height, crown length, and crown width. Furthermore, D is a primary determinant of stem volume, commonly used one-sided measures of competition [such as basal area per acre/hectare in trees with larger values of D (BAL) and crown competition factor in trees with larger values of $D$ (CCFL)] and commonly used two-sided measures of competition [such as basal area per acre/hectare (BA), crown competition factor (CCF) and stand density index (SDI)]. Therefore, the prediction of diameter increment ( $\triangle \mathrm{D}$ ) is particularly critical in tree-list models such as ORGANON (Weiskittel et al. 2011) which are used to project future values of D . If D is measured on all trees, then the other three dynamic equations can be predicted indirectly (e.g., height increment and crown recession) or from predictor variables constructed using D alone (e.g., probability of mortality using just $\mathrm{D}, \mathrm{D}^{2}$, $B A L$ and $B A$ ) if necessary.

### 2.1 Choice of the Form of the Response Variable

$\Delta \mathrm{D}$ can be used directly as the response variable or indirectly through the usage of basal area increment of the tree ( $\Delta \mathrm{ba}$ ). West (1980, p. 76) examined the use of both forms and concluded that "There seemed to be no evidence that diameter or basal area increment should, in general, be preferred in ... growth studies."

Both $\Delta \mathrm{ba}$ and $\Delta \mathrm{D}$ were tried as the response variable in the original edition of SWOORGANON (Hann and Larsen 1991), and it was decided to use $\Delta \mathrm{D}$ for two reasons. First, many of the other component equations for the southwest Oregon version of ORGANON used tree diameter rather than basal area and, as a result, it was necessary for the model to be able to project unbiased estimates of $D$ into the future. Second, transformation of the $\Delta$ ba equation to predict $\Delta \mathrm{D}$ diameter increment provided unreasonable predictions for trees with small values of $D$.

Another reason that people have exerted to justify the use of $\Delta$ ba rather than $\Delta \mathrm{D}$ as the response variable is that the application of the former produces better indices of fit. However, $\Delta$ ba is never measured directly but is instead calculated from D and $\Delta \mathrm{D}$ using the following equation:
$\Delta \mathrm{ba}=\pi / 2 \cdot\left(\Delta \mathrm{D}^{2}+2 \cdot \Delta \mathrm{D} \cdot \mathrm{D}_{\mathrm{S}}\right)$
Where,
$D_{s}$ is the measured DBH at the start of the growth period for the tree

Therefore, using $\Delta \mathrm{ba}$ as the response variable introduces $\mathrm{D}_{\mathrm{s}}$ into the response variable. Given that $D_{s}$ is also an influential predictor variable, the improved indices of fit from using $\Delta$ ba is caused by having of $D_{S}$ in both the "left" hand and "right" hand of the increment equation.

In early development of equations for predicting either $\Delta \mathrm{D}$ or $\Delta \mathrm{ba}$, the response variable was formulated as either $\ln (\Delta \mathrm{D})$ or $\ln (\Delta \mathrm{ba})$ in order to linearize the equation. The usage of this convention was driven by the fact that early linear regression programs had fewer limitations in the size of the sample and the number of predictor variables that could be used in them than the severe restrictions found with the early nonlinear regression programs. These restricts have now been eliminated so that they are no longer reasons not to use either $\Delta \mathrm{D}$ or $\Delta \mathrm{ba}$ as the response variable in a nonlinear regression program. Furthermore, usage of $\ln (\Delta \mathrm{D})$ or $\ln (\Delta \mathrm{ba})$ introduces log bias into the estimation process (Flewelling and Pienaar 1981), which is eliminated by using $\Delta \mathrm{D}$ or $\Delta \mathrm{ba}$.

### 2.2 Choice of the Basic Equation Form and Associated Predictor Variables

ORGANON uses the following general model form to characterize $\Delta \mathrm{D}$ of 19 tree species growing in untreated stands:
$\Delta \mathrm{D}=\operatorname{EXP}\left(\mathrm{b}_{0}+\mathrm{b}_{1} \times \mathrm{X}_{1}+\mathrm{b}_{2} \times \mathrm{X}_{2}+\mathrm{b}_{3} \times \mathrm{X}_{3}+\mathrm{b}_{4} \times \mathrm{X}_{4}+\mathrm{b}_{5} \times \mathrm{X}_{5}+\mathrm{b}_{6} \times \mathrm{X}_{6}\right)$
Where,

```
\(X_{1}=\ln \left(D+K_{1}\right)\)
\(\mathrm{X}_{2}=\mathrm{D}^{\mathrm{K} 2}\)
\(X_{3}=\ln \left(S I-K_{3}\right)\)
\(\mathrm{X}_{4}=\ln \left[\left(\mathrm{CR}+\mathrm{K}_{4}\right) /\left(1.0+\mathrm{K}_{4}\right)\right]\)
\(X_{5}=B A L^{K 5} / \ln \left(D+K_{6}\right)\)
\(\mathrm{X}_{6}=\mathrm{BA}^{\mathrm{K} 7}\)
\(\mathrm{D}=\mathrm{D}\) at the start of the growth period (SOG)
CR = Crown ratio of the target tree at SOG
\(\mathrm{SI}=\) Site index of the target stand
\(B A L=B A L\) at SOG
\(B A=B A\) at \(S O G\)
```

$\mathrm{b}_{0}$ to $\mathrm{b}_{6}=$ Parameters determined by nonlinear regression
$\mathrm{K}_{1}$ to $\mathrm{K}_{7}=$ Constants that are either determined from the modeling data or by convention

### 2.3 Rationales for the Form of Equation (2.1) and the $\mathrm{K}_{1}$ to $\mathrm{K}_{6}$ Constants

The $\Delta \mathrm{D}$ of a tree is influenced by the tree's size, its vigor, the level of one-sided competition for light it experiences, the level of two-sided competition for water and nutrients it experiences, and the productivity of the soil. Both ORGANON and CIPSANON use D as a measure of tree size, CR as a measure of vigor, BAL as a measure of one-sided competition for light, BA as a measure of two-sided competition for moisture and nutrients, and SI as a measure of productivity. The usage of D, CR and SI have been commonly used in tree list models (e.g., Wykoff et al. 1982, Ritchie and Hann 1985, Wensel et al. 1987, Hann and Larsen 1991, Zumrawi and Hann 1993, Hann and Hanus 2002a, Hann et al. 2003, and Hann et al. 2006). However, there has been a wider array of attributes used to describe one-sided and two-sided competition.

In the western US, there have been at least three attributes used to characterize one sided competition for light in tree list models such as ORGANON: BAL (Wykoff et al. 1982, Hann and Larsen 1991, Zumrawi and Hann 1993, Hann and Hanus 2002, Hann et al. 2003, and Hann et al. 2006), crown competition factor in trees with values of D larger than the subject tree (CCFL, Ritchie and Hann 1985), and crown closure at a percentage of the subject tree's H. In Wensel et al. 1987, crown closure was estimated at $2 / 3$ of H and, therefore, it was designated CC66, and, in Bravo et al. 2001, crown closure was estimated at the tip of the tree and, therefore, it was designated CCH. Both CC66 and CCH are estimated using crown profile equations.

Hann and Larsen (1991) examined multiple transformations of these three descriptors of onesided competition and found that one of the transformations of BAL had lower values of MSE than the others tried in that study. Bravo et al. (2001) re-examined the issue using the expanded SWO-ORGANON modeling data set and concluded that CCH was superior to BAL as a measure of one-sided competition in mixed species stands. However, the difficulty with using CCH as an indicator of one-sided competition is the necessity of having values of H and height to crown base (HCB) on all trees on a plot. Because of the prevalence of subsampling H and HCB on most research plots and forest landowner inventories, I have continued to use BAL as the indicator of one-sided competition because it uses just D, which is always measured on trees above breast height and, therefore, avoids issues with measurement error (see Canavan and Hann 2014 for a complete discussion of the consequences of having measurement error, how to test for its presence, and how to adjust or correct for the presence of measurement error).

There have been at least three "attributes" used to characterize two-sided competition for moisture and nutrients: BA (Ritchie and Hann 1985, Hann and Larsen 1991, Zumrawi and Hann 1993, Hann and Hanus 2002, Hann et al. 2003, and Hann et al. 2006), crown competition factor (CCF, Wykoff et al. 1982), and nothing (Wensel et al. 1987). While CCF has been touted as being superior to BA in mixed species stands, it proved to explain less variation in $\Delta \mathrm{D}$ than BA in the mixed species stands of southwest Oregon (Hann and Larsen 1991). As implied by the $\Delta \mathrm{D}$ equation in Wensel et al. (1987), I too have found a number of cases in which BA was not a significant contributor to the prediction of $\Delta \mathrm{D}$. This might be due to limitations in the modeling data set or to growing conditions that do not have soil moisture or nutrient deficiencies so that a one-sided competition measure is adequate.

To further understand the rationale for Equation (2.1), it is useful to reformulate the equation into the following:
$\Delta D=($ Maximum $\Delta D) \times(C R$ Modifier $) \times($ BAL Modifier $) \times($ BA Modifier $)$
Where,

```
Maximum \DeltaD = EXP{b
CR Modifier = EXP(X4)
BAL Modifier = EXP(X5)
BA Modifier = EXP(X 
```

The Maximum $\Delta \mathrm{D}$ component of Equation (2.2) uses a modification of the two-parameter Weibull probability density function (PDF) to characterize the relationship between $D$ and $\Delta D$ [i.e., $\operatorname{EXP}\left(b_{0}+b_{1} \times X_{1}+b_{2} \times X_{2}\right)$ ]. Sit and Poulin-Castello (1994) note that the Weibull function "...is very flexible, and especially useful for fitting growth and yield data." I independently came to the same conclusion in the mid 1980's (i.e., Hann and Larsen 1991).

The following form of the two parameter Weibull PDF has frequently been used in medical biometrics and econometrics:
$f(x: k, b)=b k x^{k-1} e^{-b x^{k}}$
The parameters k and b occupy a restricted numerical space that constrains the Weibull cumulative density function (CDF) to values between zero and one. This form of the Weibull PDF can be re-expressed as:
$f\left(x: a_{1}, a_{2}, a_{3}, a_{4}\right)=a_{1} x^{a_{2}} e^{a_{3} x^{a_{4}}}$
Where,
$\mathrm{a}_{1}=\mathrm{bk}$
$a_{2}=k-1$
$\mathrm{a}_{3}=-\mathrm{b}$
$\mathrm{a}_{4}=\mathrm{k}$
The modification to the Weibull PDF was to allow the $a_{1}, a_{2}, a_{3}$, and $a_{4}$ parameters to occupy a larger numerical space than that occupied by the $b$ and $k$ values of the Weibull PDF based upon the fit to the modeling data set. This modification of the Weibull PDF can produce a peak in $\Delta \mathrm{D}$ as D increases in size, as has been found in open grown trees, as long as $\mathrm{a}_{2}$ is greater than zero. The relationships between the $a_{1}, a_{2}, a_{3}$, and $a_{2}$ parameters above and the parameters in the Maximum $\Delta \mathrm{D}$ component of Equation (2.2) are:
$\mathrm{b}_{0}=\ln \left(\mathrm{a}_{1}\right)$
$\mathrm{b}_{1}=\mathrm{a}_{2}$
$\mathrm{b}_{2}=\mathrm{a}_{3}$
$\mathrm{K}_{2}=\mathrm{a}_{4}$

The addition of $\mathrm{b}_{3} \times \mathrm{X}_{3}$ to the Maximum $\Delta \mathrm{D}$ component of Equation (2.2) allows the introduction of SI into that component and, as a result allows the size of the combined intercept term (i.e., $\mathrm{b}_{0}+\mathrm{b}_{3} \times \mathrm{X}_{3}$ ) to vary with SI (which is constant for a given plot).

The remaining components of Equation (2.2) produce modifications to the modified Weibull function for the impact of $\mathrm{SI}, \mathrm{CR}, \mathrm{BAL}$, and BA upon Maximum $\triangle \mathrm{D}$. The considerable amount of flexibility in Equation (2.2) is imparted both through the regression parameters $b_{0}$ to $b_{6}$ and a set of seven constants, $\mathrm{K}_{1}$ through $\mathrm{K}_{7}$. Some of these constants are determined by convention (to be discussed below), others by a regression process that produces values with reduced number of decimal places in order to match the number of decimal places to which the predictor variable was measured, and the remainder of the constants by an iterative process to control the shape of Equation (2.1). Some of the $\mathrm{K}_{\mathrm{i}}$ constants are elements where the modeling artist can tweak to get the intended effect at the extremes of tree and stand behavior. The rationale for these seven constants follows:

1. The objective of adding $\mathrm{K}_{1}$ to D in the Maximum $\Delta \mathrm{D}$ component of Equation (2.2) is to increase predicted $\Delta \mathrm{D}$ for trees with small values of D (Hann and Larsen 1991). When $K_{2}$ is 1.0 , the addition of $K_{1}$ to $D$ shifts where the $\left(b_{0}+b_{1} \times \ln \left(D+K_{1}\right)+b_{2} \times D^{K 2}\right)$ component of Equation (2.1) predicts a zero value of Maximum $\Delta D$ at a hypothetical value of $D$ equal to negative $\mathrm{K}_{1}$ (Figure 2.1). One effect of adding $\mathrm{K}_{1}$ to D is to cause trees that are minutely below breast height at the start of the growth period (and therefore having $D=0$ ) to exhibit positive values of $\Delta D$ over the growth period. Without a positive value of $\mathrm{K}_{1}$, such trees would have a value of zero for predicted $\Delta \mathrm{D}$, and, as a result, they would never achieve a positive value of D . While the use of $\mathrm{K}_{1}$ affects only a small part of the population of trees, I feel that including $\mathrm{K}_{1}$ better characterizes the biological behavior of trees at one of the extreme values of D. The original editions of SWO-ORGANON, NWO-ORGANON, and SMC-ORGANON used a $K_{1}$ value of 1.0 in their five-year $\Delta D$ equations, as did RAP-ORGANON in both fits of its annual red alder $\triangle D$ equations. The second editions of SWO-ORGANON and SMC-ORGANON found that $\mathrm{K}_{1}$ values of 5.0 and 6.0 better characterized five-year $\Delta \mathrm{D}$. Therefore, I have found that the size of $\mathrm{K}_{1}$ is directly related to the size of the growth period. The value of $\mathrm{K}_{1}$ was limited to a single decimal point because $D$ was measured to one tenth of an inch.

Figure 2.1. Predicted maximum diameter increment for six values of $K_{1}$ using fixed values of $b_{0}, b_{1}, b_{2}$, and $b_{3}$.

2. The value of $K_{2}$ in the Maximum $\Delta \mathrm{D}$ component of Equation (2.2) was set to 2.0 for Douglas-fir in the original editions of SWO-ORGANON, NWO-ORGANON, and SMCORGANON but it was found that this value of $\mathrm{K}_{2}$ drove predicted $\Delta \mathrm{D}$ too quickly towards zero for values of $D$ that were larger than those found in the modeling data set but still within the range of $D$ values found for Douglas-fir. The addition of larger values of $D$ to the SWO-ORGANON modeling data set resulted in a change of $\mathrm{K}_{2}$ to 1.0 in the second editions of SWO-ORGANON, NWO-ORGANON, and SMC-ORGANON, and both editions of RAP-ORGANON.
3. The use of 4.5 (i.e., breast height in English units) for $\mathrm{K}_{3}$ in the SI predictor variable of the Maximum $\Delta \mathrm{D}$ component of Equation (2.2) was first introduced in the original version of SWO-ORGANON (Hann and Larsen 1991). SWO-ORGANON used the southwest Oregon dominant height growth and SI equations of Hann and Scrivani (1987), and their equations measured age at a height of $\mathrm{K}_{3}$ feet above ground. Given that both D and $\Delta \mathrm{D}$ were also measured at a height of $\mathrm{K}_{3}$ feet above the ground, it was decided to use the same value of $\mathrm{K}_{3}$ in the SI predictor variable of the SWO-ORGANON $\Delta \mathrm{D}$ equations. Both the northwest Oregon version of ORGANON and the Stand Management Cooperative version of ORGANON also used SI equations in which age was measured at $\mathrm{K}_{3}$ feet above ground. While, the red alder plantation version of ORGANON used a SI equation for red alder in which $\mathrm{K}_{3}$ was 0.0 , the SI equations for the remaining species used a value of 4.5 for $\mathrm{K}_{3}$. Regardless, the diameter increment
equations in RAP-ORGANON all used $K_{3}$ of 4.5. In hindsight, the original rationale for using $\mathrm{K}_{3}$ of 4.5 in the $\Delta \mathrm{D}$ equations seems weak, but traditions die hard.
4. The use of 0.2 for $\mathrm{K}_{4}$ in the CR Modifier of Equation (2.2) was also first introduced in the original version of SWO-ORGANON (Hann and Larsen 1991). The basal area increment ( $\Delta \mathrm{ba}$ ) equation of Ritchie and Hann (1985) used a value of 0.0 for $\mathrm{K}_{4}$. When that value was applied in the $\Delta \mathrm{D}$ equations of SWO-ORGANON, the resulting residuals showed that $\Delta \mathrm{D}$ was consistently under-estimated for small values of CR . Examination of alternative values for $\mathrm{K}_{4}$ resulted in a value of 0.2 , which both minimized the mean square error of the regression equation and removed the prediction bias for small values of CR for all of the species examined by Hann and Larsen (1991). Using a value of 0.0 for $\mathrm{K}_{4}$ has some intuitive appeal because it forces predicted $\Delta \mathrm{D}$ to be zero when CR is zero. However, trees with CR of zero are dead trees and, therefore, they are not a member of the population of interest when applying a $\Delta \mathrm{D}$ equation. The result of forcing $\Delta \mathrm{D}$ to be zero when CR is zero is biased prediction of $\Delta \mathrm{D}$ for trees with small, but positive, values of CR.
5. The value of $\mathrm{K}_{5}$ in the BAL Modifier in Equation (2.2) controls how quickly $\Delta \mathrm{D}$ will decline as BAL increases from zero. A value of 1.0 for $\mathrm{K}_{5}$ will result in a faster decline in $\Delta \mathrm{D}$ for small values of BAL than a value of 2.0. The behavior for trees with small values of BAL is particularly important because these trees are members of the dominant crown class. Therefore, they have a major impact upon stand development, both directly by being the faster growing trees in the stand and indirectly by causing the greatest amount of competition in the stand. The BAL Modifier of Equation (2.2) was first set to 2.0 for the original editions of SMC-ORGANON, NWO-ORGNON, and SMCORGANON. It was changed to 1.0 for the second editions of SMC-ORGANON, NWOORGANON, and SMC-ORGANON, and both editions of RAP-ORGANON in order to better characterize $\triangle \mathrm{D}$.
6. The inclusion of both D and $\mathrm{K}_{6}$ in the BAL Modifier of Equation (2.2) was first used in the PROGNOSIS model (which later became the Forest Vegetation Simulator or FVS). As the size of $D$ increases, the inclusion of $D$ in the BAL modifier reduces the impact upon $\Delta \mathrm{D}$ of a given level of BAL. Wykoff (1986) set $\mathrm{K}_{6}$ to 1.0 which worked without problems in PROGNOSIS because his $\Delta$ ba equation was applied only to trees with $\mathrm{D} \geq$ 10 inches. When applied in the development of the $\Delta \mathrm{D}$ equation in SWO-ORGANON, it was found that a value of 1.0 caused the BAL predictor variable to become undefined when $D$ was zero, and to unreasonably switch the sign on $b_{5}$ for $D$ values between zero and the base of the natural logarithm, e. To solve this problem requires $K_{6}$ to be at least as large as the base of the natural logarithm (e, or approximately 2.7182818 ). The value of $\mathrm{K}_{6}$ was set to 5.0 for the original editions of SMC-ORGANON, NWO-ORGANON, and SMC-ORGANON, and $\mathrm{K}_{6}$ was reset to 2.7 (the value of e rounded to one tenth of an inch) for the second editions of SMC-ORGANON and SMC-ORGANON. The first edition of RAP-ORGANON mistakenly set $\mathrm{K}_{6}$ to 1.0 , which was corrected to 2.7 in the second edition. Again, the value of $\mathrm{K}_{6}$ was limited to a single decimal point because D was measured to one tenth of an inch.
7. The value of 0.5 for $K_{7}$ in the BA Modifier of Equation (2.2) was also introduced in the southwest Oregon version of ORGANON (Hann and Larsen 1991). It proved to best characterize the relationship between BA and $\triangle \mathrm{D}$ after examining numerous alternatives across many species. One hypothesis for the superiority of 0.5 is that it best characterizes the relationship between BA and the sapwood area of the stand.

Concentrating on the Maximum $\Delta \mathrm{D}$ component of Equation (2.2), the size of $\mathrm{K}_{1}$ is probably affected by the length of the growth period and the correlations between $K_{1}$ and $b_{0}, b_{1}, b_{2}$, and $\mathrm{K}_{2}$. I would expect that the size of $\mathrm{K}_{1}$ would be smaller for annual growth periods than five-year growth periods. To examine the relationship between $K_{1}$ and $b_{0}, b_{1}, b_{2}$, and $K_{2}$ more fully, I fit Equation (2.1) to the annual $\Delta \mathrm{D}$ data that I had previously created from the SMC Type I control plots. The refit was to six combinations of $K_{1}$ and $K_{2}$ to assess their impact upon the $b_{0}, b_{1}$, and $b_{2}$ parameters and the resulting adjusted coefficient of determination $\left(\mathrm{R}_{\mathrm{a}}{ }^{2}\right)$ for each fit. This data set contained 18,919 observations, and the results are found in Table 2.1.

Table 2.1. Estimated values of $b_{0}, b_{1}$, and $b_{2}$ for Equation (2.1), and the resulting value of $R_{a}{ }^{2}$, when fitted to the SMC Type I control plot data set using alternative values of $\mathrm{K}_{1}$ and $\mathrm{K}_{2}$.

| Values of $\mathrm{K}_{1}$ and $\mathrm{K}_{2}$ | $\mathrm{~b}_{0}$ | $\mathrm{~b}_{1}$ | $\mathrm{~b}_{2}$ | $\mathrm{R}_{\mathrm{a}}{ }^{2}$ |
| ---: | ---: | ---: | ---: | ---: |
| $\mathrm{~K}_{1}=1.0, \mathrm{~K}_{2}=1$ | -5.36 | 0.70 | $-4.18 \mathrm{E}-02$ | 0.7786 |
| $\mathrm{~K}_{1}=0.1, \mathrm{~K}_{2}=1$ | -5.00 | 0.49 | $-2.51 \mathrm{E}-02$ | 0.7793 |
| $\mathrm{~K}_{1}=1.0, \mathrm{~K}_{2}=2$ | -5.24 | 0.53 | $-11.42 \mathrm{E}-04$ | 0.7778 |
| $\mathrm{~K}_{1}=0.1, \mathrm{~K}_{2}=2$ | -4.97 | 0.41 | $-6.67 \mathrm{E}-04$ | 0.7788 |
| $\mathrm{~K}_{1}=1.0, \mathrm{~K}_{2}=3$ | -5.19 | 0.49 | $-43.32 \mathrm{E}-06$ | 0.7772 |
| $\mathrm{~K}_{1}=0.1, \mathrm{~K}_{2}=3$ | -4.95 | 0.39 | $-19.97 \mathrm{E}-06$ | 0.7785 |

Examination of Table 2.1 reveals the following behavior:

1. The $R_{a}{ }^{2}$ values are very close to each other across all six alternative combinations of $\mathrm{K}_{1}$, and $\mathrm{K}_{2}$ examined in this evaluation.
2. For a fixed value of $K_{2}$, increasing $K_{1}$ resulted in smaller values of $b_{0}$ and $b_{2}$, and larger values of $b_{1}$.
3. For a fixed value of $K_{1}$, increasing $K_{2}$ resulted in slightly larger values of $b_{0}$, and smaller values of $b_{1}$ and $b_{2}$, with the changes being greater for the largest value of $K_{1}$.

These results show that it is possible to choose alternative values of $\mathrm{K}_{1}$ and $\mathrm{K}_{2}$ that can change the shape of the predictive surface of Equation (2.1)'s Maximum D component without great loss in explained variation. Such changes could prove useful in finding a model form for Equations (2.1) and (2.2) that will extrapolate reasonably well beyond the modeling data set.

### 2.4 Statistical Methods for Estimating the Parameters of Equation (2.1)

The particular model form used in Equation (2.1) was chosen to minimize parameter-effects nonlinearity, which can affect the accuracy of confidence bounds for the parameter estimates and t-tests used to test the significance of model parameters from hypothetical values (Hann et al. 1987, Ratkowski 1983). The parameter estimates are determined through application of weighted, nonlinear regression, with the reciprocal of predicted $\Delta \mathrm{D}$ as the weight. I use the transformation of both sides of the equation to estimate the weighted parameters because this process can be directly used in the nonlinear regression program NLREG, which the US Bureau of Standards has found to be more capable and accurate than SAS at solving difficult nonlinear models. The transformation process produces identical parameter estimates and their standard errors as the weighting process used in SAS and other statistical packages (Draper and Smith1998). I have found that plots of the weighted residuals from the transformation process using NLREG are better indicators of whether or not the transformation was successful at homogenizing the residuals than those produced by SAS.
The transformation process in this case involves multiplying both sides of the equation by the square root of the weight [for example, the transformed response variable would be $\Delta D /(P \Delta D)^{0.5}$ ], where $P \Delta D$ is the predicted $\Delta D$ from the previous iteration:
$\Delta D / P \Delta D^{1 / 2}=\left[E X P\left(b_{0}+b_{1} \times X_{1}+b_{2} \times X_{2}+b_{3} \times X_{3}+b_{4} \times X_{4}+b_{5} \times X_{5}+b_{6} \times X_{6}\right)\right] / P \Delta D^{1 / 2}$
Because the weight uses $P \Delta D$, estimating the weighted parameters requires the following iterative process that was described in Kmenta (1986):

1. Estimate the parameters of Equation (2.1) using unweighted regression.
2. Use the parameters from the unweighted fit to calculate $P \Delta D$ and insert it into Equation (2.3) and estimate the new weighted parameters.
3. Use the parameters from the weighted fit to recalculate $P \Delta D$ and insert it into Equation (2.3) and re-estimate the weighted parameters.
4. Repeat the third step until there are no substantial differences in the values of the parameters between the last two iterations.

While tedious, I have found that this iterative process often produces parameter estimates that exhibit a lower value of residual MSE than that produced by the reiterative refitting process found in SAS.

While weighting produces homogeneous variance of the residuals needed for inference purposes, the form of the weight gives more emphasis to trees with small increments. Whether or not one should use weighting when estimating the parameters of a model primarily intended for prediction purposes is open to debate (this is also true of the height increment equation discussed in Chapter 3.0).

### 2.5 Description of the $\Delta \mathrm{D}$ Data Sets Used in the Following Examples

Two data sets from which an edition of ORGANON was developed at two points in time during the course of data collection will be used to illustrate the effect of population definition, sampling design, and measurement specifications can have upon the parameterization of Equation (2.1). The first data set was collected in southwest Oregon and it was used to develop SWO-ORGANON. The second data set was collected using both existing research installations from members of the Stand Management Cooperative (SMC) and from new installations created by the SMC itself. This data was then used to develop SMC-ORGANON.

Details concerning the data collection methods used to collect the basic data used to model $\Delta \mathrm{D}$ in both the original analyses and the revised analyses are found in Chapter 1.0. In all four analyses, the basic data was then transformed to produce the response and basic predictor variables used in Equation (2.1) (e.g., $\Delta \mathrm{D}, \mathrm{D}, \mathrm{SI}, \mathrm{CR}, \mathrm{BAL}$. and BA).

### 2.5.1 SWO-ORGANON Data Sets

The original modeling data set contained 391 plots of which 388 plots had at least one Douglas-fir (Pseudotsuga menzziesii (Mirib.) Franco) tree that was large enough to increment bore. The original modeling data set also included all plots that had trees previously cut. The revised data set contained a combination of the 529 plots, with 526 plots containing at least one Douglas-fir that was above breast height. However, only 407 of the plots in the combined data set were not affected by cutting and contained Douglas-fir large enough to be increment bored. In addition, the change in the sampling design for the revised data set resulted in some large diameter trees being removed from the original data set. These changes removed a total of 3,028 Douglas-fir trees from the original modeling data set for $\Delta \mathrm{D}$. A summary of the descriptive statistics for the original and revised Douglas-fir modeling data sets is found in Table 2.2.

Table 2.2. Descriptive statistics for the Douglas-fir data set used to model five-year $\Delta \mathrm{D}$ in the original SWO-ORGANON analysis and the revised SWO-ORGANON analysis, $\Delta \mathrm{D}$ and D were measured in inches and BAL and BA were measured in square feet per acre.

| Attribute | Original Analysis $(\mathrm{N}=11,974)$ |  |  | Revised Analysis $(\mathrm{N}=12,403)$ |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Mean | Minimum | Maximum | Mean | Minimum | Maximum |
|  | 1.0 | 0.0 | 4.7 | 0.8 | 0.1 | 4.2 |
| D | 13.8 | 0.3 | 83.8 | 16.1 | 0.1 | 80.8 |
| CR | 0.50 | 0.02 | 1.00 | 0.46 | 0.05 | 1.00 |
| BAL | 94.6 | 0.0 | 380.0 | 110.1 | 0.0 | 516.0 |
| BA | 191.9 | 0.1 | 393.2 | 186.6 | 0.1 | 542.0 |
| SI | 93.4 | 54.1 | 141.1 | 99.6 | 41.5 | 146.9 |

### 2.5.2 SMC-ORGANON Data Sets

When the original edition of SMC-ORGANON was developed, the size of the sample used to model Equation (2.1) consisted of only 4,093 trees because of the dearth of CR measurements on silvicultural research plots at that time. The weakness of this data set resulted in the need to fix the $\mathrm{b}_{1}$ and $\mathrm{b}_{2}$ parameters to those of Hann and Larsen (1991) in order to provide biologically reasonable predictions from the resulting model (Hann et al. 2006).

The revised edition of SMC-ORGANON saw an increase to 33,094 observations available to fit Equation (2.1). A majority of the additional observations with measured CR came from three data sets collected using common sampling procedures that were first used to develop SWOORGANON (Hann and Hanus 2002), then NWO-ORGANON (Zumrawi and Hann 1993), and finally to develop a set of ORGANON like equations in southwest Washington as part of a MS degree at the University of Washington (McKenzie 1994). Unlike the measurement protocols used in silvicultural research plots (both then and currently), the ORGANON related data sets measured height and CR on all sample trees, a practice that minimizes or eliminates numerous potential problems with measurement error in the resulting growth and yield model. The remaining new trees in the modeling data set for fitting the revised equation came from more recent measurements taken on the SMC installations. A comparison of the original and revised data sets are found in Table 2.3.

Table 2.3. Descriptive statistics for the data set used to model five-year $\Delta \mathrm{D}$ in the original SMC-ORGANON analysis and the revised SMC-ORGANON analysis, $\triangle \mathrm{D}$ and D were measured in inches and BAL and BA were measured in square feet per acre.

| Attribute | Original Analysis |  |  | Revised Analysis |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Mean | Minimum | Maximum | Mean | Minimum | Maximum |
|  | 0.92 | -0.1 | 3.9 | 1.31 | 0.0 | 5.7 |
| D | 8.33 | 0.6 | 36.7 | 13.4 | 0.1 | 81.8 |
| CR | 0.53 | 0.05 | 0.92 | 0.57 | 0.04 | 1.00 |
| BAL | 85.6 | 0.0 | 365.1 | 72.6 | 0.0 | 460.0 |
| BA | 198.3 | 24.6 | 385.1 | 142.0 | 0.1 | 558.2 |

### 2.6 Example Parameterizations for $\Delta \mathrm{D}$

The original and revised parameterizations, and associated $\mathrm{K}_{\mathrm{i}}$ values, for both SWOORGANON and the SMC-ORGANON were chosen to illustrate the expected signs on the parameters and magnitude of those parameters, and the effect of different values of $\mathrm{K}_{\mathrm{i}}$ upon Equation (2.1).

### 2.6.1 SWO-ORGANON Parameters

The values of $b_{i}$ and $\mathrm{K}_{\mathrm{i}}$ for both editions of SWO-ORGANON are found in Table 2.4.

Table 2.4. The parameter estimates of Equation (2.1) for both the original edition of SWOORGANON and the revised edition of SWO-ORGANON.

| Parameter | Original SMC-ORGANON | Revised SMC-ORGANON |
| :---: | ---: | ---: |
| $\mathrm{b}_{0}$ | -3.33258 | -5.35558894 |
| $\mathrm{~b}_{1}$ | 0.401284 | 0.840528547 |
| $\mathrm{~b}_{2}$ | -0.000444053 | -0.0427481848 |
| $\mathrm{~b}_{3}$ | 0.778012 | 0.91202025 |
| $\mathrm{~b}_{4}$ | 1.34652 | 1.15950313 |
| $\mathrm{~b}_{5}$ | -0.0000496540 | -0.00894779670 |
| $\mathrm{~b}_{6}$ | -0.0151775 | 0.0 |
| $\mathrm{~K}_{1}$ | 1.0 | 5.0 |
| $\mathrm{~K}_{2}$ | 2.0 | 1.0 |
| $\mathrm{~K}_{3}$ | 4.5 | 4.5 |
| $\mathrm{~K}_{4}$ | 0.2 | 0.2 |
| $\mathrm{~K}_{5}$ | 2.0 | 1.0 |
| $\mathrm{~K}_{6}$ | 5.0 | 2.7 |
| $\mathrm{~K}_{7}$ | 0.5 | 0.5 |

A comparison of the nonlinear regression parameter estimates, $b_{0}$ to $b_{6}$, in Table 2.4 shows that both set of parameters exhibit the same signs, which is the first requirement for creating reasonable behavior of the $\Delta \mathrm{D}$ predictions from Equation (2.1) or its reformulated Equation (2.2). The prominent changes between the two parameterizations of Equation (2.2) are the values of $\mathrm{K}_{1}$ and $\mathrm{K}_{2}$ used in the maximum $\Delta \mathrm{D}$ component of Equation (2.2) and $\mathrm{K}_{5}$ and $\mathrm{K}_{6}$ used in the BAL component of Equation (2.2).

### 2.6.2 SMC-ORGANON Parameters

The first set of parameters was developed for the original edition of the Stand Management Cooperation's version of ORGANON (i.e., SMC-ORGANON, Hann et al. 2003) and the second set of parameters, and associated $\mathrm{K}_{\mathrm{i}}$ values, are those used in the revised edition of SMCORGANON (Hann et al. 2006). In incorporating the ORGANON data into the revised analysis, it was assumed that the relationship of $\Delta \mathrm{D}$ to D and $C R$ was the same across all of the modeling data sets.

Six predictor variables were added to Equation (2.1) in order to recognize differences in how, where, and when the data were collected. These additional variables identified that (1) the Hann and Scrivani (1987) SI values ( $\mathrm{SI}_{\mathrm{HS}}$ ) used in southwest Oregon differed from the Bruce (1981) SI values ( $\mathrm{SI}_{\mathrm{B}}$ ) used in all of the other data sets, (2) the calculated values of BA and BAL could be affected by the substantial difference between the ORGANON plot design and the plot design used in the other SMC data sets (Hann and Zumrawi 1991), and (3) the three ORGANON modeling data sets were collected over relatively short periods on temporary plots in different parts of the Pacific Northwest. Offsetting these issues was the substantial increase in the number of trees measured for CR and the expansion of the sample to trees with very large values of D. All of the predictor variables added to Equation (2.1) included the use of indicator (or dummy) variables (Cunia 1973) and are defined in the following manner:

$$
\begin{aligned}
& \mathrm{X}_{7}=\mathrm{I}_{\text {swo }} \\
& \mathrm{X}_{8}=\mathrm{I}_{\mathrm{NW}} \\
& X_{9}=I_{\text {WWA }} \\
& \mathrm{X}_{10}=\mathrm{I}_{\mathrm{sw}} \times \ln \left(\mathrm{SI}_{\mathrm{HS}}-\mathrm{K}_{3}\right) \\
& \mathrm{X}_{11}=\mathrm{I}_{\mathrm{ORG}} \times\left\{\mathrm{BAL} /\left[\ln \left(\mathrm{D}+\mathrm{K}_{6}\right)\right]\right\} \\
& \mathrm{X}_{12}=\mathrm{l}_{\mathrm{ORG}} \times\left(\mathrm{BA}^{\mathrm{K7}}\right) \\
& I_{\text {Swo }}=1.0 \text { if data came from the SWO-ORGANON data set, }=0.0 \text { otherwise } \text {. } \\
& I_{\mathrm{NWO}}=1.0 \text { if data came from the NWO-ORGANON data set, }=0.0 \text { otherwise } \text {. } \\
& I_{\text {WWA }}=1.0 \text { if data came from the WWA-ORGANON data set, }=0.0 \text { otherwise } \text {. } \\
& l_{\text {IRG }}=I_{\text {SWO }}+I_{\text {NWO }}+I_{\text {wWA }}
\end{aligned}
$$

In order to remain congruent with the definition of $X_{10}, X_{4}$ was redefined as:

$$
\mathrm{X}_{4}=\left(1.0-\mathrm{I}_{\mathrm{SWO}}\right) \times \ln \left(\mathrm{SI}_{\mathrm{B}}-\mathrm{K}_{3}\right)
$$

The indicator variables $l_{\text {org }}, I_{\text {swo }}, I_{\text {Iwo }}$, and $I_{\text {wwa }}$ were all set to 0.0 when Equation (2.1) was inserted into the revised version of SMC-ORGANON, which resulted in the elimination of predictor variables $\mathrm{X}_{7}$ to $\mathrm{X}_{12}$.

The increase in the sample size of the revised modeling data set led to the ability to fit revised Equation (2.1) which, in turn, resulted in parameter estimates which met biological expectations. The values of $b_{i}$ and $K_{i}$ for both editions of SMC-ORGANON are found in Table 2.5.

Table 2.5. The parameter estimates of Equation (2.1) for both the original edition of SMCORGANON and the revised edition of SMC-ORGANON, with the indicator variables loRg, $I_{\text {swo }} I_{\text {Nwo, }}$ and $I_{\text {wwa }}$ all set to 0.0 for the revised edition of SMC-ORGANON.

| Parameter | Original SMC-ORGANON | Revised SMC-ORGANON |
| :---: | ---: | ---: |
| $\mathrm{b}_{0}$ | -5.252294703 | -5.34253119 |
| $\mathrm{~b}_{1}$ | 0.401284000 | 1.09840684 |
| $\mathrm{~b}_{2}$ | -0.000444053 | -0.05218621 |
| $\mathrm{~b}_{3}$ | 1.19147443 | 0.91202025 |
| $\mathrm{~b}_{4}$ | 1.056689 | 1.0138081 |
| $\mathrm{~b}_{5}$ | -0.000048600 | -0.0175622 |
| $\mathrm{~b}_{6}$ | -0.016648482 | -0.05168923 |
| $\mathrm{~K}_{1}$ | 1.0 | 6.0 |
| $\mathrm{~K}_{2}$ | 2.0 | 1.0 |
| $\mathrm{~K}_{3}$ | 4.5 | 4.5 |
| $\mathrm{~K}_{4}$ | 0.2 | 0.2 |
| $\mathrm{~K}_{5}$ | 2.0 | 1.0 |
| $\mathrm{~K}_{6}$ | 5.0 | 2.7 |
| $\mathrm{~K}_{7}$ | 0.5 | 0.5 |

A comparison of the nonlinear regression parameter estimates, $b_{0}$ to $b_{6}$, in Table 2.5 shows that both set of parameters exhibit the same signs, which is the first requirement for creating
reasonable behavior of the $\Delta \mathrm{D}$ predictions from Equation (2.1) or its reformulated Equation (2.2). The prominent changes between the two parameterizations of Equation (2.2) are the values of $K_{1}$ and $K_{2}$ used in the maximum $\Delta D$ component of Equation (2.2) and $K_{5}$ and $K_{6}$ used in the BAL component of Equation (2.2).

### 2.7 Visually Examining the Predictions of $\Delta \mathrm{D}$ from the Equations

Because of the complexity of Equation (2.1), I find it useful to also graphically examine the behavior of the equation. I further find it useful to use the components expressed in Equation (2.2) as a basis for creating the graphs. Each of the three modifiers in Equation (2.2) predicts relative reductions in the rates of $\Delta \mathrm{D}$ that vary from zero (resulting in no increment) to 1.0 (resulting in maximum increment). Two sets of example graphs are presented below in order to show the amount of variability that can occur from fitting Equation (2.1) to different data sets. In reviewing these graphs and the equations behind them, it should be remembered that, while the equations meet biological expectations, there are portions of their multidimensional prediction surface in which trees in real life conditions would not be found to occupy [such as an 80 -inch tree growing in a stand with a SI value of 40 -feet, or an 80 -inch tree with a BAL of 400 square feet per acre, etc., though Equation (2.1) will predict values of $\Delta \mathrm{D}$ for these conditions and these predictions can be used in preparing graphs].

### 2.7.1 Original SWO-ORGANON $\Delta \mathrm{D}$ Equation versus Revised SWO-ORGANON $\triangle \mathrm{D}$ Equation

Two comparisons will be made in order to examine the impact of the alternative data sets upon the resulting parameters and their predictions: the original SWO-ORGANON $\triangle \mathrm{D}$ parameters versus the revised SWO-ORGANON $\triangle \mathrm{D}$ parameters. Figure 2.2 shows the maximum $\Delta \mathrm{D}$ that both the original SWO-ORGANON and the revised SWO-ORGANON predict for three values of SI plotted across D when CR is set to one, BAL is set to zero, and BA is set to zero. The resulting predictions of maximum $\Delta \mathrm{D}$ are simulations of the $\Delta \mathrm{D}$ that would be expected for open grown trees (with the exception that the tree's basal area per acre is not included in the BA modifier).

Figure 2.2. Maximum predicted $\Delta \mathrm{D}$ from the original SWO-ORGANON equation and the revised SWO-ORGANON for three values of SI plotted across D when CR is set to one, BAL is set to zero, and BA is set to zero.


Figure 2.2 shows that the relative shape of the original and revised five-year equations both meet expected behavior across D and SI . However, the revised five-year $\Delta \mathrm{D}$ equation does predict higher peak values of maximum $\Delta \mathrm{D}$ at a D value of approximately 15 -inches, instead of the approximately 20 -inches in the original analysis, and that the revised five-year $\Delta \mathrm{D}$ equation also predicts higher values of maximum $\Delta \mathrm{D}$ at a D value of 80 -inches, which can be attributed to the effect of including data from trees with large values of $D$ in the revised analysis.

It should be noted that both equations shown in Figure 2.2 exhibit the property that $\Delta \mathrm{D}$ peaks at the same D value for each of the three site indices. Previous work has indicated that the peak in $\Delta \mathrm{D}$ should occur at increasingly younger ages as SI increases. Figure 2.3 shows $\Delta \mathrm{D}$ plotted over breast height age for both the original SMC-ORGANON and the revised SMCORGANON equations

Figure 2.3. Maximum predicted $\Delta \mathrm{D}$ from the original SWO-ORGANON equation and the revised SWO-ORGANON for three values of SI plotted across breast height age when CR is set to one, BAL is set to zero, and BA is set to zero.


Figure 2.3 shows that both equations exhibit the expected behavior but that the original equation's peaks in $\Delta \mathrm{D}$ occur at unreasonably large values of breast height age. Another interesting observation about Figure 2.3 is that $\Delta \mathrm{D}$ of open grown trees is predicted to be nearly the same across SI after a breast height age of approximately 60 years. However, this shared value of $\Delta \mathrm{D}$ is being added to trees with much larger values of D on high values of SI than low values of SI, and, as a result, the basal area increment remains much larger on high SI sites than low SI sites after 60 years.

As we will discuss next, these potential growth rates will be modified by CR and competition which will also be impacted by the fitting data sets so that overall predictions are generally consistent for each data set.

The relative reductions in $\Delta \mathrm{D}$ arising from original SWO-ORGANON equation and the revised SWO-ORGANON are plotted across CR in Figure 2.4.

Figure 2.4. CR modifiers to Maximum $\Delta \mathrm{D}$ for both the original SWO-ORGANON and revised SWO-ORGANON $\Delta \mathrm{D}$ equations.


CR is a strong indicator of tree vigor, with more vigorous trees growing faster than the less vigorous trees. The results in Figure 2.4 shows that both the revised and original equations predict relative growth rates in $\Delta \mathrm{D}$ to strongly increase with increasing CR , as expected. Figure 2.4 also shows that the revised equation will predict somewhat larger values of $\Delta \mathrm{D}$ than the original equation for smaller values of CR.

The relative reductions in $\Delta \mathrm{D}$ arising from applying both the original SMC-ORGANON $\Delta \mathrm{D}$ equation and the revised SMC-ORGANON $\triangle \mathrm{D}$ equation to Maximum $\triangle \mathrm{D}$ are plotted across BAL in Figure 2.5 for three values of $D(1 ", 10 "$, and 20 ").

Figure 2.5. BAL modifiers to maximum $\Delta \mathrm{D}$ for the original SMC-ORGANON $\Delta \mathrm{D}$ equation and the revised SMC-ORGANON $\triangle$ D equation for three values of $D\left(1^{\prime \prime}, 10^{\prime \prime}\right.$, and 20 ").


The results in Figure 2.5 show that both the revised and original modifier equations predict relative growth rates in $\Delta \mathrm{D}$ to strongly decrease with increasing BAL, as expected. Further inspection of Figure 2.5 shows how the change in $\mathrm{K}_{5}$ from 2.0 in the original SWO-ORGANON BAL Modifier to 1.0 in the revised SWO-ORGANON modifier has changed the shape of the modifier from one which predicts a relative growth rate that decreases slowly across low values of BAL to one that predicts a much quicker change in relative growth rate. For a given value of $D$, the original modifier consistently predicts smaller reductions in $\Delta D$ than the revised modifier for values of BAL up to approximately 400 square feet per acre.

The relative reductions in $\Delta \mathrm{D}$ arising from both the original SWO-ORGANON $\triangle \mathrm{D}$ equation and the revised SWO-ORGANON $\Delta \mathrm{D}$ equation BA modifiers to the Maximum $\Delta \mathrm{D}$ are plotted across BA in Figure 2.6.

Figure 2.6. BA modifiers to maximum $\Delta \mathrm{D}$ for the original SWO-ORGANON and revised SWOORGANON $\triangle$ D equations.


Figure 2.6 show that only the original equation predicts relative growth rates in $\Delta \mathrm{D}$ to decrease with increasing BA, as expected. The fact that the revised Equation (2.1) is not affected by BA could be the result of the change to the BAL modifier between the original and revised equations. BA can be re-expressed as
$B A=B A L+B A S$
Where,
BAS = Basal area per acre in trees with values of $D$ equal to or smaller than the subject tree's D

The BAL modifiers in Figure 2.5 show that the revised modifier has a stronger negative impact upon $\Delta \mathrm{D}$ than the original modifier. Therefore, if the revised BAL modifier is a better descriptor of the impact of BAL upon $\triangle \mathrm{D}$ than the original modifier, its usage could be reducing the effect of the BAL component in the BA modifier leaving the contribution of BAS as the primary competitive effect in the BA modifier. Hann (1980) found that the negative impact upon basal area increment of ponderosa pine resulting from BA in larger trees was more than four times greater than BA in smaller trees. This implies that a statistically significant impact of BAS upon
increment could be more difficult to detect, particularly from data collected with probability proportional to size (which would result in more sampling variation for small trees). As a result, a small parameter and a large standard error of the parameter could result in the parameter testing to be statistically insignificant.

### 2.7.2 Original SMC-ORGANON $\Delta \mathrm{D}$ Equation versus Revised SMC-ORGANON $\Delta \mathrm{D}$ Equation

Two comparisons will be made in order to examine the impact of alternative data sets upon the resulting parameters and their predictions: the original SMC-ORGANON $\triangle$ D parameters versus the revised SMC-ORGANON $\triangle$ D parameters with the indicator variables lorg, $I_{\text {swo }} I_{\text {nwo }}$, and $I_{\text {wwa }}$ all set to 0.0 . Figure 2.7 shows the maximum $\Delta \mathrm{D}$ that both the original SMC-ORGANON and the revised SMC-ORGANON predict for three values of SI plotted across D when CR is set to one, BAL is set to zero, and BA is set to zero. The resulting predictions of maximum $\Delta \mathrm{D}$ are simulations of the $\Delta \mathrm{D}$ that would be expected for open grown trees (with the exception that the tree's basal area per acre is not included in the BA modifier).

Figure 2.7. Maximum predicted $\triangle \mathrm{D}$ from the original SMC-ORGANON equation and the revised SMC-ORGANON for three values of SI plotted across D when CR is set to one, BAL is set to zero, and BA is set to zero.


Figure 2.7 shows that the relative shape of the original and revised five-year equations both meet expected behavior across D and SI . However, the revised five-year $\Delta \mathrm{D}$ equation does predict higher peak values of maximum $\Delta \mathrm{D}$ at a D value of approximately 15 -inches, instead of the approximately 21 -inches in the original analysis, and that the revised five-year $\Delta \mathrm{D}$ equation
also predicts higher values of maximum $\Delta \mathrm{D}$ at a D value of 80 -inches, which can be contributed to the effect of including data from trees with large values of $D$ in the revised analysis.

Two methods of measuring HCB have been used extensively in the Pacific Northwest. In the first method, the lower branches on the longer side of the crown for trees of uneven crown length are transferred mentally to fill in the missing portion of the shorter side of the crown. The objective of this method is to generate a "full, even crown". HCB is then measured to this mentally generated position on the bole (epicormic and short internodal branches are ignored). This method is used in the collection of the three ORGANON related data sets and in the collection of data on research plots that predated the SMC.

In the second method, crown base is defined as the lowest whorl with live branches in at least three quadrants around the stem circumference. Again, epicormic branches and whorls not continuous with the main crown are ignored. The HCB by this method $\left(\mathrm{HCB}_{3 / 4}\right)$ is the distance from the ground to the whorl defining this crown base. Maguire and Hann (1987) showed that $\mathrm{HCB}_{3 / 4}$ was greater than or equal to HCB. Because $\mathrm{HCB}_{3 / 4}$ is the method used in the SMC installation data sets, the equation of Hann and Hanus (2002a) was used to convert $\mathrm{HCB}_{3 / 4}$ to HCB. This conversion equation predicts very small differences between $\mathrm{HCB}_{3 / 4}$ and HCB for trees with very large CR. Therefore, the correction was small for the young, long-crowned trees in the Type I and III data sets used to develop the original and revised editions of SMCORGANON.

The relative reductions in $\Delta \mathrm{D}$ arising from original SMC-ORGANON equation and the revised SMC-ORGANON are plotted across CR in Figure 2.8.

Figure 2.8. CR modifiers to Maximum $\Delta \mathrm{D}$ for both the original SMC-ORGANON and revised SMC-ORGANON $\Delta \mathrm{D}$ equations.


CR is a strong indicator of tree vigor, with more vigorous trees growing faster than the less vigorous trees. The results in Figure 2.8 shows that both the revised and original equations predict relative growth rates in $\Delta \mathrm{D}$ to strongly increase with increasing CR, as expected. Figure 2.8 also shows that the revised equation will predict somewhat larger values of $\Delta \mathrm{D}$ than the original equation for smaller values of CR.

The relative reductions in $\Delta \mathrm{D}$ arising from applying both the original SMC-ORGANON $\Delta \mathrm{D}$ equation and the revised SMC-ORGANON $\triangle \mathrm{D}$ equation to maximum $\triangle \mathrm{D}$ are plotted across BAL in Figure 2.9 for three values of $D(1 ", 10 "$, and 20 ").

Figure 2.9. BAL modifiers to maximum $\Delta \mathrm{D}$ for the original SMC-ORGANON $\triangle \mathrm{D}$ equation and the revised SMC-ORGANON $\triangle$ D equation for three values of $D\left(1^{\prime \prime}, 10^{\prime \prime}\right.$, and 20 ").


The results in Figure 2.9 show that both the revised and original modifier equations predict relative growth rates in $\Delta \mathrm{D}$ to strongly decrease with increasing BAL, as expected. Further inspection of Figure 2.9 shows how the change in $\mathrm{K}_{5}$ from 2.0 in the original SMC-ORGANON BAL Modifier to 1.0 in the revised SMC-ORGANON modifier has changed the shape of the modifier from one which predicts a relative growth rate that decreases slowly across low values of BAL to one that predicts a much quicker change in relative growth rate. For a given value of D , the original modifier consistently predicts smaller reductions in $\Delta \mathrm{D}$ than the revised modifier for values of BAL up to approximately 400 square feet per acre.

The relative reductions in $\triangle \mathrm{D}$ arising from both the original SMC-ORGANON $\triangle \mathrm{D}$ equation and the revised SMC-ORGANON $\triangle \mathrm{D}$ equation BA modifiers to the Maximum $\Delta \mathrm{D}$ are plotted across BA in Figure 2.10.

Figure 2.10. BA modifiers to maximum $\Delta \mathrm{D}$ for the original SMC-ORGANON and revised SMCORGANON $\triangle$ D equations.


Figure 2.10 show that both the revised and original equations predict relative growth rates in $\Delta \mathrm{D}$ decrease with increasing BA, as expected. As with the BAL modifiers, the predicted values of relative growth rate from the original SMC-ORGANON BA modifier are higher than the revised SMC-ORGANON BA modifier.

These comparisons show that the revised SMC-ORGANON $\triangle \mathrm{D}$ equation will predict higher $\triangle \mathrm{D}$ for open grown trees and for trees with smaller values of CR than the original SMCORGANON $\triangle \mathrm{D}$ equation. These increases are counteracted by the larger decreases in $\Delta \mathrm{D}$ as both BAL and BA increases for the revised equation. The differences between the original SMC-ORGANON $\triangle \mathrm{D}$ equation and revised SMC-ORGANON $\triangle \mathrm{D}$ are the result of both changes in the $\mathrm{K}_{\mathrm{i}}$ values for the two equations and by the increase in the amount and extent of modeling data available to the fit the revised equation.

### 2.8 Final Observations

Differences between the revised SMC-ORGANON $\triangle \mathrm{D}$ equation and the revised SWO $\Delta \mathrm{D}$ equation are the result of the different type of SI equations, definitions of the target populations, and sampling designs used to collect the modeling data. Advantages of fixed area
permanent plots that are used to collect the data on research installations, such as installed by the SMC, include:

1. Repeat measurements over longer time periods reduces the impact of variations in the weather
2. Treatments, and their effect upon tree development, are easier to conduct on fixed area plots with repeat measurements
3. The rate of mortality is more accurately estimated on fixed area plots with repeat measurements
4. Larger samples of $\Delta \mathrm{H}$ and $\triangle \mathrm{HCB}$ are created over time

Advantages of the sampling design and measurement protocols used to collect SWO, NWO, and SWW modeling data in the targeted populations of operational stands include:

1. All trees were measured for H and HCB and, as a result, H and HCB measures of competition could be calculated without injecting measurement error into their calculation
2. Data was collected in operational stands, the ultimate application of the resulting model
3. Application to operational stands can result in the measurement of a greater range in stand structures and species composition
4. The sampling design used to collect the SWO, NWO, and SWW modeling data is closer to that used in operational inventories than the fixed area plots commonly used in research installations, reducing problems with measurement errors in the predictor variables when the resulting model is applied to data from inventories (Canavan and Hann 2014).
5. The intensity of sampling (i.e., the number of plots installed per acre) in the target population was greater than that found in all permanent research installations in the region that were designed to study stand development
6. The sampling design targeted the larger trees on a plot which are the largest valued, fastest growing, and most competitive trees on a plot.
7. The use of temporary plots relatively quickly provided modeling data in areas and/or stand structures and species compositions devoid of permanent research installations

Regardless of how the modeling data is collected, Equation (2.1) has been successfully applied and then used to make biologically reasonable projections of $\Delta \mathrm{D}$ for numerous species across many areas in the Pacific Northwest and the southern USA. Therefore, parameter estimates for Equation (2.1) that test to be insignificantly different from zero indicate problems with the modeling data and NOT the model form of Equation (2.1).

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### 3.0 Height Increment Equation ( $\Delta \mathrm{H}$ )

Modeling height increment $(\Delta \mathrm{H})$ is generally more difficult than $\Delta \mathrm{D}$ because of higher withinstand variability in $\Delta H$ due to larger measurement errors in H as H increases (Larsen et al. 1987), a more limited number of repeat measurements, and a closer connection to environmental factors rather than stand-level factors (Weiskittel et al. 2011).

### 3.1 Basic Equation

ORGANON uses the following general model form to characterize $\Delta H$ of multiple tree species growing in untreated stands:
$\Delta \mathrm{H}=\mathrm{P} \Delta \mathrm{H} \times \mathrm{MOD}$
Where,
$\mathrm{P} \Delta \mathrm{H}=$ Potential height increment of the subject tree as if it were a site height tree
MOD $=P \Delta H$ modifier equation
MOD $=b_{1} \times\left\{\mathrm{X}_{1}+\left(\mathrm{X}_{2}-\mathrm{X}_{1}\right) \times E X P\left[\mathrm{X}_{3} \times(1.0-\mathrm{CR})^{2}\right]\right\}$
$\mathrm{X}_{1}=\mathrm{b}_{2} \times \mathrm{EXP}\left(\mathrm{b}_{3} \times \mathrm{CCH}\right)$
$\mathrm{X}_{2}=\operatorname{EXP}\left(\mathrm{b}_{4} \times \mathrm{CCH}^{\mathrm{K}}\right)$
$\mathrm{X}_{3}=\mathrm{b}_{5} \times \mathrm{EXP}\left(\mathrm{b}_{6} \times \mathrm{CCH}^{\mathrm{K}}\right)$
$\mathrm{CCH}=$ Crown closure at the tip of the subject tree in percent
$\mathrm{CR}=$ Crown ratio of the subject tree
$\mathrm{K}=0.5$ or 1.0
The form of Equation (3.2) was the result of an extensive evaluation of alternative model forms and predictor variables. From this process, it was found that several measures of crown closure at various heights on the subject trees were the only one-sided predictor variables which had parameter estimates that were significantly different from zero. Examination of alternative heights on a tree for estimating crown closure found that the best place for explaining the most variation in $\Delta \mathrm{H}$ was at the tip of the tree (i.e., CCH ).

It should be noted that $\mathrm{P} \Delta \mathrm{H}$ is the only tree attribute to have a unit of measure in Equation (3.1). CR is the ratio of crown length divided by H , and CCH is expressed as a percentage. Therefore, the parameters of Equation (3.2) can be directly compared between equations developed using English units of measure and those developed using metric units of measure.

To calculate $\mathrm{P} \Delta \mathrm{H}$ requires both the height $(\mathrm{H})$ of the subject tree at the start of the growth period and the SI of the plot using dominant height growth and SI equation(s) selected for the species of interest. These are then used to determine the tree's growth effective age (GEA) at the start of the growth period from the dominant height growth equation. This is done by determining the age the subject tree would have to be in order to have a dominant height of value H for the plot of interest with the given value of SI (e.g., Hann and Ritchie 1988). Potential height $(\mathrm{PH})$ at the end of the growth period is then predicted for the combination of

GEA+GP (where GP is the length of the prediction growth period) and SI. Finally, $\mathrm{P} \Delta \mathrm{H}$ is calculated as PH minus H .

The following limits on each of the parameter estimates are the minimum required to produce expected behavior from Equation (3.1):

1. Parameter $b_{1}$ should be greater than or equal to 1.0 and less than or equal to 1.1
2. Parameter $b_{2}$ should be greater than or equal to 0.0 and less than or equal to 1.0
3. Parameters $b_{3}, b_{4}$, and $b_{5}$ should be less than 0.0
4. Parameter $b_{6}$ should be greater than or equal to 0.0

Meeting each of these limits is necessary but insufficient to guarantee reasonable behavior from Equation (3.1).

A tree (or multiple trees if more than one has the same value of H ) with a CCH of zero is the tallest tree on the plot and, therefore, it belongs in the population of top height trees. For a tree with a CR equal one and a value of CCH equal to zero, $\mathrm{b}_{1}$ indicates the proportion of $\mathrm{P} \Delta \mathrm{H}$ that the tree(s)'s $\Delta H$ will experience in the next growth period.

The size of $b_{1}$ will, in part, be determined by the method used to develop and then apply the dominant height growth equation that is used to predict $\mathrm{P} \Delta \mathrm{H}$. The two most frequently used methods for estimating SI are through the usage of SI equations developed using either the permanent-plot method or stem-analysis method (Hann 1995). The permanent-plot method uses age and top height (e.g., the 40 largest diameter trees per acre, H 40 ) on multiple plots that were remeasured for age and H 40 over time to develop the SI equation. The stemanalysis method uses a relatively small sample of site quality trees that have been (usually) felled and then sectioned in order to determine their breast height ages at selected heights at and above breast height. Therefore, the primary difference is that the permanent-plot method uses stand age and H 40 to determine SI and the stem-analysis method uses tree age and tree height to determine the SI of one or more site quality trees on the plot. As a result, the permanent plot method is usually applied to evenaged stands while the stem analysis method can be applied to both evenaged and unevenaged stands. These differences can, therefore, impact the size of $b_{1}$.

If the permanent-plot method was used to develop and then apply the $\mathrm{SI} /$ dominant-heightgrowth (SI/DHG) equation, then the $b_{1}$ parameter should be greater than one in order for the resulting predicted average H (for those trees forming the H 40 population on the plot at the end of the growth period) to be equal to that predicted from the $\mathrm{SI} / \mathrm{DHG}$ equation at the end of the growth period. It can be particularly disconcerting to the users of the model if the predicted value of H 40 at the base age of the SI equation is not equal to the value of the SI for the plot being simulated. While this approach does not guarantee that the average H of the H 40 trees on the plot will be equal to that predicted from the SI/DHG equation, one must remember that the two estimates of H 40 at a given age are both from a measurement/sampling process that contains error. Values of $b_{1}$ that are near to or less than 1.0 might indicate that measurements of SI in the modeling data set are somewhat too high.

If the stem analysis method was used to develop the $\mathrm{SI} / \mathrm{DHG}$ equation, then it is possible that the value of $b_{1}$ will be closer to one because of the number and selection method used to measure SI/DHGSI/dominant-height-growth. The common method used to estimate SI from of a stem-analysis equation is to select between five to six trees per acre using a specified plot design, measure their heights and breast height ages, calculate SI for each tree, and average the tree-level SI values to obtain the plot's value of SI . The trees chosen are frequently the tallest undamaged trees from the dominant or codominant crown class and, as a result, they often exhibit the highest $\Delta \mathrm{H}$ values on the plot. If the procedures defined by the SI/DHG equation recommends using 1/5-acre plots (e.g., Barrett 1978 and Cochran 1979) to estimate SI , then the result is to measure only one tree on the plot, and, as a result, it should be expected that $b_{1}$ will close, if not equal, to 1.0 . This would also be true of the six trees per acre recommendation of Hann and Scrivani (1987).

In summary, the permanent-plot method uses the average height of 40 largest trees per acre based upon D, along with plot age, to calculate the plot's SI , while the stem-analysis method uses the average site index of the largest 5 to 6 trees per acre based upon height. Therefore, the value of $b_{1}$ should exceed one by a greater amount if the SI/DHG equation used to predict $\mathrm{P} \Delta \mathrm{H}$ was developed using the permanent-plot method rather than the stem-analysis method.

### 3.2 Rationale for the Form of Equation (3.1)

The concept behind Equation (3.1) is to first determine $\mathrm{P} \Delta \mathrm{H}$ for the subject tree and then to use Equation (3.2) to modify P $\mathrm{P} H$ for the subject tree's $C R$ and $C C H$. It is expected that $P \Delta H$ will decrease with decreasing CR and increasing CCH. To achieve this expectation, Equation (3.2) was formed as sigmoidal in shape when CR increases from zero to one for a given value of CCH , and, for a given value of CR , each CR sigmoid will decrease with a decrease in CCH .

With CCH fixed to a particular value of interest, the inflection point of the sigmoid is defined by $X_{3}$, the minimum value of the sigmoid is defined by $X_{1}$, and the maximum value of the sigmoid is defined by $\mathrm{X}_{2}$.

For CCH of zero, Equation (3.2) simplifies to:

$$
\begin{equation*}
\mathrm{MOD}_{\mathrm{CCH}=0}=\mathrm{b}_{1} \times\left\{\mathrm{b}_{2}+\left(1.0-\mathrm{b}_{2}\right) \times E X P\left[b_{5} \times(1.0-C R)^{2}\right]\right\} \tag{3.3}
\end{equation*}
$$

Equation (3.3) further simplifies to the following when CR is zero:

$$
\begin{equation*}
\mathrm{MOD}_{\mathrm{CR}=0, \mathrm{CCH}=0}=\mathrm{b}_{1} \times\left\{\mathrm{b}_{2}+\left(1.0-\mathrm{b}_{2}\right) \times \mathrm{EXP}\left(\mathrm{~b}_{5}\right)\right\} \tag{3.4}
\end{equation*}
$$

Conversely, Equation (3.3) further simplifies to the following when CR is one:

$$
\begin{equation*}
\mathrm{MOD}_{\mathrm{CR}=1, \mathrm{CCH}=0}=\mathrm{b}_{1} \tag{3.5}
\end{equation*}
$$

Equations (3.4) and (3.5) can be useful in further evaluating the reasonableness of the estimated parameters of Equation (3.1).

In most parameterizations of Equation (3.1), predicted $\Delta \mathrm{H}$ does not go to zero when CR is zero.
Hann and Ritchie (1988) note that, while it is true that trees with no CR (i.e., dead trees) should have zero $\Delta \mathrm{H}$, the objective of Equation (3.1) is to predict the $\Delta \mathrm{H}$ of living trees, and the vast majority of living trees in a stand have CRs > than 0.15 . As examples, there are three G\&Y data sets in the Pacific Northwest in which CR was measured on all trees: those trees measured for development of SWO-ORGANON, NWO-ORGANON, and a data set measured in western Washington (WWA) by professor Douglas A. Maguire while he was at the University Washington. The percentage of Douglas-fir trees measured in those data sets with CRs $\leq 0.15$ was $1.7 \%$ in SWO, $0.5 \%$ in NWO, and $0.8 \%$ in WWA. Therefore, constraining $\Delta H$ to go to zero when CR is zero is not necessary and its use unduly distorts the response surface. For those concerned about this feature of Equation (3.1), ORGANON has an additional modifier that drives predicted $\Delta \mathrm{H}$ to zero when CR is $<0.15$.

### 3.3 Statistical Methods for Estimating the Parameters of Equation (3.2)

The six parameters of Equation (3.2) are determined by fitting the following equation to the modeling data set using unweighted, nonlinear regression:
$\Delta \mathrm{H} / \mathrm{P} \Delta \mathrm{H}=\mathrm{b}_{1} \times\left\{\mathrm{X}_{1}+\left(\mathrm{X}_{2}-\mathrm{X}_{1}\right) \times \mathrm{EXP}\left[\mathrm{X}_{3} \times(1.0-\mathrm{CR})^{2}\right]\right\}$
Use of the ratio $\Delta H / P \Delta H$ as the response variable helps to homogenize the variance about predicted $\Delta \mathrm{H}$. This method of transforming the response variable to homogenize the variance will produce identical parameter estimates and their standard errors as applying weighted regression to Equation (3.1) with a weight of $\mathrm{P} \Delta \mathrm{H}^{-2}$ (e.g., Cunia 1964, Hann et al. 1987, Draper and Smith 1998). I prefer this method of estimating the weighted statistics of a regression equation. This is because the resulting weighted residuals are straight forward to calculate and, to me, they are easier to interpret than those produced by some regression packages. Having good initial estimates of the parameters are critical in successfully fitting Equation (3.6) to the modeling data set. I have found that the values of the parameters from an earlier fit of Equation (3.6) can usually serve as good initial estimates.

### 3.4 Description of the $\Delta H$ Data Sets Used in the Following Examples

Two data sets from which an edition of ORGANON was developed at two points in time over the course of data collection will be used to illustrate the effects that population definition, sampling design, and measurement specifications can have upon the parameterization of Equation (3.1). The first data set was collected as part of two research projects in southwest Oregon and it was used to develop SWO-ORGANON. The second data set was collected at two consecutive times using both existing research installations donated by members of the Stand Management Cooperative (SMC) and from new installations created by the SMC itself. This data was then used to develop SMC-ORGANON.

Details concerning the data collection methods used to collect the basic data used to model $\Delta \mathrm{H}$ in both the original analyses and the revised analyses are found in Chapter 1.0. In both analyses, the basic data was then transformed to produce the response and basic predictor variables used in Equation (3.1) (e.g., $\Delta \mathrm{H}, \mathrm{P} \Delta \mathrm{H}, \mathrm{CR}$, and CCH ).

### 3.4.1 SWO-ORGANON Data Sets

The original modeling data set contained 391 plots of which 388 plots had at least one Douglas-fir tree. The original modeling data set also included all plots that had seen previously cutting of trees. Of the 529 plots in the combined data set, 526 plots contained at least one Douglas-fir that was above breast height. However, only 407 of the plots in the combined data set were not affected by cutting. In addition, the change in the sampling design for the revised data set resulted in some large diameter trees being removed from the original data set. These changes removed a total of 262 Douglas-fir trees from the original modeling data set for $\Delta \mathrm{H}$. A summary of the descriptive statistics for the original and revised Douglas-fir modeling data sets is found in Table 3.2.

Table 3.2. Descriptive statistics for the data set used to model five-year $\Delta \mathrm{H}$ in the original SWO-ORGANON analysis and the revised SWO-ORGANON analysis, $\Delta \mathrm{H}$ and H were measured in feet and CCH in percent. The reduction in the number of $\Delta \mathrm{H}$ observations in the revised analysis was due to the elimination of 262 observations due to their being affected by recent cutting.

| Attribute | Original Analysis $(\mathrm{N}=2483)$ |  | Revised Analysis $(\mathrm{N}=2436)$ |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Mean | Minimum | Maximum | Mean | Minimum | Maximum |
| $\Delta \mathrm{H}$ | 6.4 | 0.3 | 18.7 | 5.1 | 0.1 | 17.5 |
| H | 45.1 | 4.6 | 203.2 | 39.0 | 4.6 | 203.2 |
| CR | 0.67 | 0.01 | 1.00 | 0.56 | 0.05 | 1.00 |
| CCH | 27.5 | 0.0 | 174.1 | 73.6 | 0.0 | 336.0 |

The largest difference between the data sets described in Table 3.2 is the near tripling in the size of mean CCH from the original to the revised edition. This change is a direct result of the addition of the old growth stands in the revised data set. The modest reductions in mean $\Delta \mathrm{H}, \mathrm{H}$ and CR could be the result of eliminating the 262 trees from stands that had received recent cutting.

### 3.4.2 SMC-ORGANON Data Sets

When the original edition of SMC-ORGANON was developed, the size of the sample used to parameterize Equation (3.1) consisted of only 3,200 trees because of the dearth of CR measurements on silvicultural research plots at that time. The revised edition of SMCORGANON saw an increase to 6,430 observations available to fit Equation (3.1). A comparison of the original and revised data sets is found in Table 3.2.

Table 3.2. Descriptive statistics for the data set used to model five-year $\Delta H$ in the original SMC-ORGANON analysis and the revised SMC-ORGANON analysis, $\Delta \mathrm{H}, \mathrm{H}$, and $\mathrm{SI}_{\mathrm{B}}$ were measured in feet and CCH in percent.

|  | Original Analysis $(\mathrm{N}=3200)$ |  |  | Revised Analysis $(\mathrm{N}=6430)$ |  |  |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Attribute | Mean | Minimum | Maximum | Mean | Minimum | Maximum |
| $\Delta \mathrm{H}$ | 9.6 | 0.2 | 30.7 | 12.5 | 0.2 | 27.8 |
| H | 52.3 | 7.0 | 147.9 | 26.2 | 4.6 | 140.9 |
| CR | 0.63 | 0.09 | 0.97 | 0.80 | 0.09 | 1.00 |
| CCH | 23.3 | 0.0 | 364.4 | 10.9 | 0.0 | 364.4 |
| $\mathrm{SI}_{\mathrm{B}}$ | 115.0 | 77.6 | 142.0 | 128.0 | 75.2 | 187.2 |

Examination of Table 3.2 shows that the revised data had the average value of H that was $50 \%$ smaller than the original data set, the average value of CCH decreasing by $53 \%$ from the original data set, and average value of CR increasing by $27 \%$ from the original data set. These combined values mask the fact that the data from just the SMC installations had an average H of 19.7 feet compared to an average H of 47.4 feet for the non-SMC data, the average value of CCH for the SMC data was $2.5 \%$ compared to an average CCH of $38.3 \%$ for the non-SMC data, and the average value of CR for the SMC data was 0.88 compared to an average CR of 0.56 for the non-SMC data.

### 3.5 Examining $\mathbf{\Delta H}$ Parameterizations of Interest

The parameterizations of two of the versions of ORGANON (the Stand Management Cooperative's version and the southwest Oregon version) will be examined to illustrate the expected signs on the parameters and the magnitude of those parameters for Equation (3.1). Furthermore, two sets of parameters will be examined for each of these two versions of the simulator: the original set of parameters and a revised set of parameters resulting from fitting Equation (3.1) to expanded data sets.

### 3.5.1 SWO-ORGANON

The model form of Equation (3.1) was created in the original edition of SWO-ORGANON using the multidimensional graphical techniques developed by Jensen (1984). This technique provided a clear view of the response surface, the resulting model form and initial parameter estimates. Limitations of the nonlinear regression routine available at that time resulted in two of the six parameters being fixed to the initial parameter estimates from the graphical method.

In the revised edition of SWO-ORGANON, Equation (3.1) was refit using the expanded data set, and, by this time, the capability of the nonlinear regression routine had improved to the point such that all six parameters could be estimated simultaneously. The resulting original (Hann and Ritchie 1988) and revised (Hann and Hanus 2002) parameter estimates for Equation (3.1) are found in Table 3.3.

Table 3.3. The parameter estimates for the revised SWO-ORGANON $\Delta H$ equation (parameter estimates arising from the graphical method are noted with asterisks).

| Parameter | Original SWO-ORGANON | Revised SWO-ORGANON |
| :---: | ---: | ---: |
| $b_{1}$ | $1.0644^{*}$ | 0.92140706 |
| $b_{2}$ | 0.876948 | 1.0 |
| $b_{3}$ | -0.0365001 | -0.02457621 |
| $b_{4}$ | -0.0506209 | -0.00407303 |
| $b_{5}$ | $-1.5625^{*}$ | -2.89556338 |
| $b_{6}$ | 0.136986 | 0.0 |
| $K$ | 0.5 | 1.0 |

The $\mathrm{b}_{1}$ parameter is a data set specific adjustment upon $\mathrm{P} \Delta \mathrm{H}$ when CCH is zero. Therefore, it should not be significantly less from 1.0 if the dominant height growth equations used to form $\mathrm{P} \Delta \mathrm{H}$ are appropriate for the species and location. Examination of Table 3.3 shows that $\mathrm{b}_{1}$ was smaller than 1.0 for the revised SWO-ORGANON model, and application of a t-test showed that the difference was statistically significant. This result indicates that $\Delta \mathrm{H}$ for trees with CCH of zero were significantly smaller than the value of $\mathrm{P} \Delta \mathrm{H}$ expected for the five growth periods (i.e., 1976 to 1981, 1977 to 1982, 1978 to 1983, 1991 to 1995 , and 1992 to 1996) that were measured in Hann and Hanus (2002). In addition, the $\mathrm{b}_{2}$ parameter was found to be not significantly different from one and $b_{6}$ parameter was not significantly different and zero, so they were set to those values in the revised edition of SWO-ORGANON.

The Hann and Scrivani (1987) SI/DHG equation used to form $\mathrm{P} \Delta \mathrm{H}$ was developed from a subset of the felled trees used in Hann and Hanus (2002). Furthermore, the equations had been validated on an independent data set (Hann 1998). Therefore, it is unlikely that the significant difference of $b_{1}$ from 1.0 for the revised edition of SWO-ORGANON indicated a problem with the SI/DHG equation used to form $\mathrm{P} \Delta \mathrm{H}$. Wensel and Turnblom (1998) and Yeh and Wensel (2000) had shown that precipitation and temperature differences between growth periods can have a significant effect upon the growth rates of trees in northern California. These factors could possibly explain the results found in Hann and Hanus (2002).

To explore this possibility further, Hann and Hanus (2002) identified the felled Douglas-fir trees with CCH of zero that had been used in the development of the dominant height growth equations of Hann and Scrivani (1987), and the ratio of $\Delta \mathrm{H} / \mathrm{P} \Delta \mathrm{H}$ was calculated for each tree. The mean of this ratio was 0.909 for the six trees meeting the selection criteria. This result was similar to the value of $b_{1}$ in Table 3 and, therefore, it indicated that, for the growth periods measured in this study, $\Delta \mathrm{H}$ was lower than the average long-term growth rates determined from stem analysis of the dominant, site quality Douglas-fir trees used in Hann and Scrivani (1987). Therefore, it was decided to set $b_{1}$ to 1.0 when used in the revised edition of SWOORGANON.

### 3.5.2 SMC-ORGANON

The first set of SMC-ORGANON parameters was developed for the original edition of the version (Hann et al. 2003) and the second set of parameters are those used in the revised
edition of the version (Hann et al. 2006). The changes in the size and composition of modeling data set that were discussed in Section 3.4.2 did have an impact upon the resulting values of the parameter estimates, as shown in Table 3.4.

Table 3.4. The parameter estimates for the original SMC-ORGANON and the revised SMCORGANON $\Delta H$ Equation (3.1).

| Parameter | Original SMC-ORGANON | Revised SMC-ORGANON |
| :---: | ---: | ---: |
| $b_{1}$ | 1.052301385 | 1.010018427 |
| $b_{2}$ | 0.638569239 | 0.655258886 |
| $b_{3}$ | -0.005328221 | -0.006322913 |
| $b_{4}$ | -0.049351159 | -0.039409636 |
| $b_{5}$ | -0.464049843 | -0.597617316 |
| $b_{6}$ | 0.485384235 | 0.631643636 |
| $K$ | 0.5 | 0.5 |

A comparison of the parameter estimates in Table 3.4 shows that both sets of parameters exhibit the same signs, which is the first requirement for reasonable behavior of the $\Delta \mathrm{D}$ predictions from Equation (3.1).

I have found that the CR value in which predictions from Equation (3.3) drops below one is related to the tolerance of the species to shading. For example, $a b_{2}$ value of one in Equation (3.3) will result in $\Delta H=b_{1} \times P \Delta H$ for trees with CCH of zero, regardless of CR. Ritchie and Hann (1990) found that the value of Equation (3.3) dropped below one more quickly as CR declined for ponderosa pine, an intolerant species, than the more tolerant species they studied. For the SMC editions of ORGANON, Equation (3.3) is predicted to cross one at a CR value of approximately 0.45 for the original SMC-ORGANON equation and a CR value of approximately 0.75 for the revised edition of SMC-ORGANON equation, the latter being influenced by the low value of $b_{1}$.

Applying Equation (3.4) with the parameters found in Table 3.4 results in modifier values of 0.911095590 for the original SMC-ORGANON equation and 0.85337288 for the revised SMCORGANON equation when both CR and CCH are zero. Using Equation (3.5) results in a modifier value of 1.052301385 for the original SMC-ORGANON equation and a modifier value of 1.010018427 for the revised SMC-ORGANON equation when CR is one and CCH is zero. Remember, Bruce's (1981) SI/dominant-height-growth equation was used to estimate $\mathrm{P} \Delta \mathrm{H}$ and, therefore, it is expected that $\mathrm{b}_{1}$ should be greater than one in order to provide reasonable values of H for the top height trees.

### 3.6 Visually Examining the Predictions of $\Delta H$ from the Equations

As $C C H$ increases from zero and $C R$ decreases from one, predicted $\Delta H$ will decline from $P \Delta H$. The rate of this decline is best examined graphically. Three example graphs are presented below in order to show the amount of variability that can occur from fitting Equation (3.1) to different data sets. In reviewing these graphs and the equations behind them, it should be remembered that, while the equations meet biological expectations, there are portions of their multidimensional prediction surface in which trees would not occupy.

### 3.6.1 SWO-ORGANON

The SWO $\Delta \mathrm{H}$ modeling data set was constrained by the fact that only a small subset of the measured trees on each plot were selected for tree felling and the measurement of $\Delta \mathrm{H}$. In addition, the definition of the population of interest changed substantially by the addition of both the hardwood dominated stands and the old growth stands and the elimination of the recently cut data. To examine the effect of these changes upon the predicted $\Delta \mathrm{H}$ modifier values for both the original and the revised editions of SWO-ORGANON, the two equations were graphed across crown ratio for three values of $\mathrm{CCH}: 0 \%, 20 \%$ and $100 \%$ (Figure 3.1).

Figure 3.1. Predicted Douglas-fir $\Delta H$ modifier equations for the original and revised editions of SWO-ORGANON equations plotted over CR for CCH values of 0.0\%, 20.0\%, and 100.0\%.


The revised SWO-ORGANON modifier equation predicts lower values of relative increment for CCH of zero percent and CR greater than 0.35 but higher values of relative increment for CCH greater than zero percent. For CCH values greater than zero, the revised SWO-ORGANON
modifier equation predicts greater values of relative increment than the original SWO-
ORGANON equation. These differences between editions could be due to the expansion of the modeling data set to other stand structures, the usage of $\mathrm{K}=0.5$ in the original edition and $\mathrm{K}=$ 1.0 in the revised edition, and/or to the improved capability of the nonlinear regression program that was available for fitting the revised edition of SWO-ORGANON.

### 3.6.2 SMC-ORGANON

The predicted $\Delta \mathrm{H}$ modifier values for both the original SMC-ORGANON equation and the revised SMC-ORGANON equation were graphed across crown ratio for three values of CCH: 0\%, 20\% and 100\% (Figure 3.2).

Figure 3.2. Predicted Douglas-fir $\Delta H$ modifier equations for the original SMC-ORGANON equation and the revised SMC-ORGANON equation plotted over CR for CCH values of $0.0 \%$, $20.0 \%$, and $100.0 \%$.


Figure 3.2 shows that the revised SMC-ORGANON $\Delta H$ modifier equation consistently predicts smaller $\Delta H$ than the original equation. The data used for the revised equation differs from the original modeling data set through the addition of more data from the SMC installations than was available in the original analysis (Table 3.2). Therefore, the more than doubling of the sample size for the revised modeling data set came from smaller trees with resulting longer crowns and experiencing lower levels of one-sided competition than the mostly older trees in the original modeling data set. I suspect that this change in the structure of the data may have caused the reduction of $b_{1}$ from 1.052301385 for the original SMC-ORGANON equation to 1.010018427 for the revised equation. My conclusion is that adding so much small, young tree data to the larger, older data in the original modeling data set did little to improve characterization of the modifier equation.

This conclusion comes after finding a similar problem with parameterization of Equation (3.1) in the CIPSANON model. The development of a good quality model requires both biologically meaningful model forms and good quality data to parameterize them. Both SMC-ORGANON and RAP-ORGANON were developed using modeling data from research installations in which both H and HCB were subsampled. This subsampling often resulted in problems such as concentrating the sample in undamaged trees and not sampling all top height trees. These, and other, problems would be resolved with the ending of subsampling for H and HCB on research installations. This was the SMC's measurement protocol when professor Maguire ran the cooperative but it ended when he left the institution.

### 3.6.3 SMC-ORGANON vs SWO-ORGANON

A comparison of the revised edition of SMC-ORGANON versus the revised edition of SWOORGANON is shown in Figure 3.3.

Figure 3.3. Predicted Douglas-fir $\Delta H$ modifier equations for the revised editions of SWOORGANON and SMC-ORGANON equations plotted over CR for CCH values of 0.0\%, 20.0\%, and $100.0 \%$.


These two versions of ORGANON illustrate how: (1) the type of stand structures chosen for the population of interest, (2) the size and design of the sample plots used to collect the modeling data set from that population, and (3) the different definitions of the basic measurements used to create the predictor variables can impact the size and precision of the
parameters in Equation (3.2). The SMC-ORGANON version's data came from fixed area, evenaged research plots installed in pure Douglas-fir, while the SWO-ORGANON version's data came from an inventory style grid of variable radius plots that were installed in operational evenaged, two-storied, and unevenaged ${ }^{1}$ stands with often multiple tree species in the stand. As result, the SWO data set contained more within stand variability in stocking (Hann and Hanus 2002), species mix and age mix than the SMC data set. This gave the SWO data set a greater set of local environments within the stand for the Douglas-fir trees to develop and, as a result, greater range in CR for a given value of CCH, which helps to explain the increase in the divergence between the SMC-ORGANON and the SWO-ORGANON modifiers shown in Figure 3.3 as CCH increases. Other factors affecting the difference in Equation (3.2) between the two versions include:

1. The usage of $\mathrm{K}=1.0$ for the SWO version and $\mathrm{K}=0.5$ for the SMC version
2. The usage of different definitions of CR
3. The usage of different types of SI/DHG equations
4. Subsampling of H and HCB for the SMC version versus complete measurement of H and HCB for the SWO version, which resulted in the introduction of measurement error into the estimates of CCH for the SMC version (see Canavan and Hann 2014 for a complete discussion of the consequences of having measurement error, how to test for its presence, and how to adjust or correct for the presence of measurement error)

### 3.7 An Alternative Equation

Equation (3.1) was designed with the objective of keeping the average predicted height of the top height trees as close as possible to that predicted by the SI/DHG equation used to calculate $\mathrm{P} \Delta \mathrm{H}$. Imposing a restriction such as this usually comes at a cost of lower precision in predicting $\Delta \mathrm{H}$. The following, alternative model form does not impose such a restriction:
$\Delta H=E X P\left[a_{0}+a_{1} \times Z_{1}+\left(a_{2,0}+a_{2,1} \times\left. S\right|^{0.5}\right) \times Z_{2}+a_{3} \times Z_{3}+a_{4} \times Z_{4}+a_{5} \times Z_{5}\right]$
Where,

$$
\begin{aligned}
& \mathrm{Z}_{1}=\ln (\mathrm{H}) \\
& \mathrm{Z}_{2}=\mathrm{H}^{\sqrt{1}} \\
& \mathrm{Z}_{3}=\ln (\mathrm{SI}) \\
& \mathrm{Z}_{4}=\ln \left[\left(\mathrm{CR}+\mathrm{J}_{2}\right) /\left(1.0+\mathrm{J}_{2}\right)\right] \\
& \mathrm{Z}_{5}=\mathrm{CCH} \\
& \mathrm{a}_{0} \text { to } \mathrm{a}_{5}=\text { Parameters determined by nonlinear regression } \\
& J_{1} \text { to } J_{3}=\text { Constants that are either determined from the modeling data or by convention }
\end{aligned}
$$

Equation (3.7) is based upon the model form used for predicting $\Delta \mathrm{D}$ that has been adapted to use the same predictor variables incorporated in Equation (3.1), including H and SI used to calculate $\mathrm{P} \Delta \mathrm{H}$. To minimize measurement errors resulting in the use of predicted instead of measured values of CR and CCH, Equation (3.7) was fit using the revised SWO-ORGANON data set that, for simplicity, has been modified by removing all data from all of the plots that

[^0]had received prior cutting and all plots without direct measurements of Douglas-fir SI (as a result, it differs somewhat from the data set used to fit Equation (3.1) in SWO-ORGANON).

Several alternative values of $J_{1}$ and $J_{3}$ were tried using this data set and in both cases a value of 1.0 was found to reduce residual MSE more than the alternatives. Alternative values of $J_{2}$ were also tried and the same value used in the $\Delta \mathrm{D}$ equation (i.e., 0.2 ) was judged to be best for Equation (3.7). It was also found that the fit to the modeling data was improved by changing the parameter on $Z_{2}$ to be a function of the square root of SI. To aid in making comparisons, Equation (3.1) was also fit to this somewhat modified data set.

The resulting adjusted coefficient of determination for the unweighted fit to Equation (3.1) was 0.7116 and the adjusted coefficient of determination for the unweighted fit to Equation (3.7) was 0.7626 . Therefore, the cost of using a model form that restricts predicted top height values is a reduction of 0.0510 in the adjusted coefficient of determination. The weighted parameter estimates for the two equations are found in Table 3.5.

Table 3.5. The weighted parameter estimates for Equation (3.1) and Equation (3.7) fit to the modified revised SWO-ORGANON $\Delta H$ modeling data.

| Parameter | Equation (3.1) | Parameter | Equation (3.7) |
| :---: | ---: | :---: | ---: |
| $\mathrm{b}_{1}$ | 0.879653093 | $\mathrm{a}_{0}$ | -1.13213812 |
| $\mathrm{~b}_{2}$ | 1.0 | $\mathrm{a}_{1}$ | 0.482546019 |
| $\mathrm{~b}_{3}$ | -0.0185901051 | $\mathrm{a}_{2,0}$ | -0.0304266413 |
| $\mathrm{~b}_{4}$ | -0.00373853371 | $\mathrm{a}_{2,1}$ | 0.00199188312 |
| $\mathrm{~b}_{5}$ | -3.8108188 | $\mathrm{a}_{3}$ | 0.938312209 |
| $\mathrm{~b}_{6}$ | 0.0 | $\mathrm{a}_{4}$ | 0.504751176 |
| K | 1.0 | $\mathrm{a}_{5}$ | -0.00490202819 |
|  |  | $\mathrm{~J}_{1}$ | 1.0 |
|  |  | $\mathrm{~J}_{2}$ | 0.2 |
|  | $\mathrm{~J}_{3}$ | 1.0 |  |

Examination of the parameters for Equation (3.7) in Table 3.5 indicates that the signs on all parameters meet expectations. The values of $\mathrm{a}_{1}, \mathrm{a}_{2,0}$, and $\mathrm{a}_{2,1}$ indicate that maximum $\Delta \mathrm{H}$ occurs at H values of 27.1, 38.3, 56.1, and 92.2 -feet for SI values of 40,80 , 120, and 160 -feet, respectively. A graph of predicted five-year height increment of Douglas-fir for SI values of 40, 80, 120, and 160-feet from both the DHG equation of Hann and Scrivani (1987) and Equation (3.7) in which CR was fixed at 0.667 and CCH was fixed at 0.0 is found in Figure 3.4.

Figure 3.4. Predicted five-year height increment of Douglas-fir for SI values of 40, 80, 120, and 160 -feet from both the dominant height equation of Hann and Scrivani (1987) and Equation (3.7) in which CR was fixed at 0.667 and CCH was fixed at 0.0.


Examination of Figure 3.4 shows that the sizes of predicted five-year height increment for the two equations are approximately the same for the four values of SI , with the best agreement occurring at the central SI values of 80 and 120 -feet. The biggest differences between the two equations occur where the peak values are located. The peak values of predicted five-year height increment from the Hann and Scrivani (1987) height increment equation occur at smaller values of H than that found for Equation (3.7) (e.g., approximately 19 vs 27.1, 28 vs $38.3,41$ vs 56.1 , and 55 vs 92.2 -feet for SI values of $40,80,120$, and 160 -feet, respectively). The CR assumption of a constant value of 0.667 that was used for predicting Equation (3.7) in preparing Figure 3.4 is not realistic. CR values of dominant trees would usually start near one at young ages and decrease to values ranging from $1 / 3$ to $2 / 3$, depending upon stand density, as the stand matures. However, inserting such realism in Equation (3.7) would require the usage of a fully developed growth and yield model, which was not available when preparing Figure 3.4. These results indicate that it is possible for a model form such as used in Equation
(3.7) to produce reasonable predictions of top height over time, but that this outcome is not as likely as using Equation (3.1).

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### 4.0 Probability of Mortality Equation (PM)

Mortality is the process which releases resources that allows surrounding trees to survive and grow. Of the four dynamic equations used in growth models, mortality remains one of the most challenging to develop due to its apparent stochastic nature and episodic occurrence (Weiskittel et al. 2011). Even "regular' competition induced mortality can be sporadic where stressed trees barely survive until AN event, such as a droughty period, occurs precipitating a wave of mortality. The mortality rate equations in ORGANON predict regular, or density dependent, mortality that is caused primarily by competition-induced suppression. However, higher amounts of stand density can also result in slow growth rates, which can weaken the defenses of the tree and making it more susceptible to mortality due damaging agents such as insects (Weiskittel et al. 2011). Determining whether the tree died as a result of competition or to insect attack is difficult to determine.

### 4.1 Basic Equation

ORGANON uses the following logistic model form to characterize the annual or five-year probability of mortality (PM) of multiple tree species growing in untreated stands:

$$
\begin{equation*}
P M_{G P}=\left[1.0+e^{-Z}\right]^{-1} \tag{4.1}
\end{equation*}
$$

Where,
GP = Length of the prediction growth period
$\mathrm{Z}=$ The logistic link function (Flewelling and Monserud 2002)
$Z=a_{0}+a_{1} \times D+a_{3} \times C R+a_{4} \times B A L+a_{5} \times S$ Sl $_{\text {HS }}$ for original SWO-ORGANON
$Z=a_{0}+a_{1} \times D+a_{2} \times D^{2}+a_{3} \times C R+a_{4} \times B A L+a_{5} \times S I_{H S}+a_{6} \times B A L \times E X P\left(a_{7} \times O G\right)$ for revised SWO-ORGANON (Z.2)
$Z=b_{0}+b_{1} \times D+b_{2} \times P C R+b_{4} \times B A L+b_{5} \times S I_{B}$ for original SMC-ORGANON
$Z=b_{0}+b_{1} \times D+b_{3} \times X C R+b_{4} \times B A L+b_{5} \times S I_{B}$ for revised SMC-ORGANON
$\mathrm{D}=$ Diameter at breast height in inches
CR = Measured crown ratio
$B A L=$ Basal area per acre in trees with D values larger than the subject tree
$\mathrm{SI}_{\mathrm{HS}}=$ Site index in feet predicted from the Hann and Scrivani (1987) equation
PCR = Predicted CR calculated from predicted HCB and predicted H for the target tree
$\mathrm{OG}=\mathrm{D} 5 \times \mathrm{H} 5 / 10,000$, which has been found to be a good indicator of old growth
D 5 = Average D of the five trees with the largest D values on the plot
$\mathrm{H} 5=$ Average H of the five trees with the largest D values on the plot
XCR = CR when it is measured and PCR when it was not measured
$S I_{B}=$ Site index in feet predicted from the Bruce (1981) equation
4.2 Rationale for the Form of Equation (4.1)

Hamilton (1986) listed the following advantages of the logistic model:

1. "The function is bounded by zero and one. Thus, the function is naturally limited to the potential range of probabilities of mortality.
2. "With the selection of the proper set of variables and transformations, the logistic function can be used to describe most naturally occurring patterns of mortality.
3. "Nonlinear estimation routines can efficiently estimate the parameters of the logistic function. This efficiency does not appear to be affected by the selection of starting values."

I have examined numerous tree and stand attributes and numerous transformations of these attributes used to form predictor variables in the $Z$ function of Equation (4.1). These predictor variables can be grouped into five classes: (1) tree size, (2) tree vigor, (3) tree position in the stand, (4) stand productivity, and (5) stand density. It is expected that PM will: (1) initially decrease and then increase with increasing tree size, (2) increase with decreasing tree vigor, (3) increase with deteriorating tree position in the stand, (4) increase with increasing stand productivity, and (5) increase with increasing stand density.

The choice of the particular predictor variables to include in a PM equation is partially determined based upon the measurement protocols used to collect the data. For example, Bravo et al. (2001) found that CCH was a better tree position variable than BAL for predicting PM. To avoid issues with measurement error, CCH requires the measurement of H on all trees, and BAL requires the measurement of D on all trees. Given that most research studies measure D on all trees but H on only a subset of trees, the usage of CCH for predicting PM is more problematic than the usage of BAL because of the introduction of measurement error when H is estimated rather than measured (see Canavan and Hann 2014 for a complete discussion of the consequences of having measurement error, how to test for its presence, and how to adjust or correct for the presence of measurement error).

Similarly, the usage of PCR or XCR instead of CR in PM models is forced upon the modeler when CR is not measured on each sample tree, which was the case with the SMC modeling data sets. The usage of XCR assumes that the subsample of trees in which CR is measured were selected in a random fashion on each plot. When the subsample is not random, then the relationship between CR and PCR may be biased on the plot and, therefore, brings into question the validity of using XCR. In this case, it is probably best to use PCR for the vigor related variable on all sample trees instead of the mix of CR and PCR found in XCR, though PCR is most likely to be less successful as a predictor of PM than CR.

Over the years, my work on predicting PM has settled upon D as a measure of size, CR (or PCR or XCR) as a measure of tree vigor, BAL as a measure of tree position within the stand, and SI as a measure of productivity. Surprisingly, I have not found a situation in which a measure of stand density improved the prediction of PM. I have also found that no transformations of these tree and stand attributes nor the usage of predicted $\Delta \mathrm{D}$ that materially improved the prediction of PM using the logistic function.

I have also found that the inclusion of data from old growth stands used in the revised SWOORGANON required the addition of both $D^{2}$ and a second BAL related variable (Hann and

Hanus 2001). The addition of $D^{2}$ was necessary to predict the expected increase in PM for large trees. The second $B A L$ related variable [i.e., $\mathrm{b}_{6} \times \mathrm{BAL} \times \mathrm{EXP}\left(\mathrm{B}_{7} \times \mathrm{OG}\right)$ ] was added in recognition that old stands are more open in structure than young stands and, therefore, a certain amount of BAL has less negative impact upon PM for trees growing in old stands than trees growing in young stands.

### 4.3 Statistical Methods for Estimating the Parameters of Equation (4.1)

The variable sizes of the growth periods lengths (LEN) in the SMC modeling data sets required that the parameters be estimated using the following formulation (Flewelling and Monserud 2002) in order to standardize them to five-year predictions (in which case LEN would be the number of five-year growth periods for each measurement in the modeling data set and, as a result, LEN could be a fractional number):

$$
\begin{equation*}
P S=\left[1.0+e^{z}\right]^{-L E N} \tag{4.2}
\end{equation*}
$$

Where,
PS = The five-year probability of survival, which was estimated by using a dichotomous survival variable (SURV) in which $\operatorname{SURV}=1.0-$ MORT.

The need to use Equation (4.2) instead of Equation (4.1) to estimate the parameters in the Zfunction is due to the fact that survival is a Markov process which allows the usage of LEN as an exponential in Equation (4.2), while mortality is not a Markov process (a tree can die only once in a given growth period but a tree can survive for multiple growth periods, Flewelling and Monserud 2002). It should be noted that the only difference between Equation (4.1) and Equation (4.2) is that Equation (4.1) has a negative sign on the Z-functions and Equation (4.2) does not. In this formulation, the parameters estimated using Equation (4.2) can be used in Equation (4.1) without any changes.

Flewelling and Monserud (2002) also provide an excellent summary of the alternative methods for estimating the parameters in the Z-functions, and for alternative statistics that can be used to evaluate the resulting fits to the data set. Application of the alternative procedures for estimating the parameters of the $Z$-function are straight forward if a common value of LEN across plots is found in the modeling data set and this value of LEN will be used as growth period (GP) in the resulting model (as was done for SWO-ORGANON).

However, the process becomes more difficult if there are multiple values of LEN in the modeling data set that differ substantially from the GP to be used in the model (as was the case for SMC-ORGANON), or if there are multiple values of LEN in the modeling data set and the GP value was one (which was the case for the RAP version of ORGANON). In the situation where the value of LEN for the modeling data set differed substantially from the value of GP to be used in the model, Flewelling and Monserud (2002) suggested that the range of LEN values be restricted to a narrow interval about the model's intended value of GP. Therefore, the values of LEN for the modeling data set used in developing the PM equation for SMC-

ORGANON were limited to between three and seven years, which kept the values of LEN for the modeling data close to the GP of five used by SMC-ORGANON.

The modeling process becomes even more complicated if it is desired to predict annual probability of mortality from data collected using varying values of LEN. Flewelling and Monserud (2002) present four methods for addressing this situation (in declining order of their preference):

1. A simulation method that was specific to one of the data sets that they were using.
2. A method analogous to that proposed by Cao (2000) which linearly interpolates the predictor variables between the start of the measurement growth period and the end of the measurement growth period.
3. A method that uses the average of the values of the predictor variables at the start and end of the growth period as an estimate of those predictor variables at the center of the growth period.
4. Simply using the predictor variables at the start of the variable length growth periods.

The first three methods assume that the mortality trees had, as a minimum, their values of $D$ measured at both the start and end of the growth period in which they died.

One of the data sets they used in their evaluations of alternative methods of fitting annual PM equations was the control plot data for unthinned SMC plots. They reported that method (3) performed worse than method (4) for that data set. When I examined their parameters, I found more change in the parameters between those two methods than between the other two methods examined. All of the predictor variables that they used in their PM models were functions of $D$. An examination of the SMC Type I data disclosed that $D$ was not always measured at the end of the growth period on dead trees, but Flewelling and Monserud (2002) did not mention if they recognized the problem and, if they did, how they addressed it. These problems could explain why method (4) performed better on the SMC Type I data than method (3).

The lack of measurements at the end of the growth period for trees that died in the growth period is not unusual. For example, the Hardwood Silviculture Cooperative's red alder data set has no measurements at the end of the growth period for dead trees. The lack of measurements on the predictor variables at the end of the growth period for some or all dead trees leaves only method (4) for estimating annual mortality rate.

The regression coefficients for Equation (4.2) used in the original editions of SWO-ORGANON (Hann and Wang 1990) and SMC-ORGANON (Hann et al. 2003) were estimated using RISK (Hamilton 1974), a program useful when the capabilities of computers were very modest (Flewelling and Monserud 2002). In the most recent, revised editions of SWO-ORGANON (Hann and Hanus 2001) and SMC-ORGANON analysis (Hann et al. 2006), the regression coefficients were estimated by using the maximum likelihood estimation procedures of SAS. The dichotomous survival variable was used as the dependent variable.

### 4.4 Description of the PM Data Sets Used in the Following Examples

Two data sets from which an edition of ORGANON was developed at two points in time over the course of data collection will be used to illustrate the effect of population definition, sampling design, and measurement specifications can have upon the parameterization of Equation (4.1). The first data set was collected in southwest Oregon and it was used to develop SWO-ORGANON. The second data set was collected using both existing research installations from members of the Stand Management Cooperative (SMC) and from new installations created by the SMC itself. This data was then used to develop SMC-ORGANON.

Details concerning the data collection methods used to collect the basic data used to model PM in both the original analyses and the revised analyses are found in Chapter 1.0. In all four analyses, the basic data was then transformed to produce the dichotomous response (e.g., 1.0 if a tree died in LEN and 0.0 if it did not die) and basic predictor variables used in Equation (4.1) (e.g., D, SI, CR, BAL, and OG).

### 4.4.1 SWO-ORGANON Data Sets

The original modeling data set contained 391 plots of which 389 plots had at least one Douglas-fir tree. The original modeling data set also included all plots in which trees had been previously cut. Of the 529 plots in the combined data set, 527 plots contained at least one Douglas-fir tree. However, only 407 of the plots in the combined data set were not affected by cutting. In addition, the change in the sampling design for the revised data set resulted in some large diameter trees being removed from the original data set. A summary of the descriptive statistics for the original and revised Douglas-fir modeling data sets is found in Table 4.1.

Table 4.1. Descriptive statistics for the Douglas-fir data set used to model five-year PM in the original SWO-ORGANON analysis and the revised SWO-ORGANON analysis, D was measured in inches, BAL was measured in square feet per acre, SI $_{H S}$ was measured in feet, and OG was measured in (inches $\times$ feet)/10,000.

| Attribute | Original Analysis $(\mathrm{N}=11,974)$ |  |  | Revised Analysis $(\mathrm{N}=17,271)$ |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Mean | Minimum | Maximum | Mean | Minimum | Maximum |
|  | 12.2 | 0.1 | 83.8 | 13.9 | 0.1 | 80.8 |
| CR | 0.48 | 0.01 | 1.00 | 0.46 | 0.02 | 1.00 |
| BAL | 101.9 | 0.0 | 380.0 | 133.9 | 0.0 | 542.0 |
| SI $_{\text {HS }}$ | 93.1 | 54.1 | 141.1 | 98.9 | 41.5 | 146.9 |
| OG | NA | NA | NA | 0.330 | 0.003 | 1.523 |

### 4.4.2 SMC-ORGANON Data Sets

When the original edition of SMC-ORGANON was developed, the size of the sample used to model Equation (4.1) consisted of mostly plots without measured values of CR. This led to the decision to use only PCR in Equation (4.1).

The revised edition of SMC-ORGANON saw an increase in the number of plots with measured values of CR. As a result, it was decided to use XCR in the fit of the revised data set to Equation (4.1). The new trees in the revised modeling data set used in fitting the revised equation came from more recent measurements taken on the SMC installations. A comparison of the original and revised data sets is found in Table 4.2.

Table 4.2. Descriptive statistics for the data set used to model five-year PM in the original SMC-ORGANON analysis and the revised SMC-ORGANON analysis, D was measured in inches, BAL was measured in square feet per acre, and $\mathrm{SI}_{\mathrm{B}}$ was measured in feet.

| Attribute | Original Analysis |  |  | Revised Analysis |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Mean | Minimum | Maximum | Mean | Minimum | Maximum |
|  | 7.1 | 0.1 | 67.1 | 6.4 | 0.1 | 67.1 |
| PCR | 0.48 | 0.13 | 0.97 | NA | NA | NA |
| XCR | NA | NA | NA | 0.54 | 0.13 | 1.00 |
| BAL | 115.1 | 0.0 | 400.2 | 97.6 | 0.0 | 400.2 |
| SIB $^{2}$ | 111.2 | 56.1 | 156.0 | 116.9 | 56.1 | 182.7 |

### 4.5 Example Parameterizations for PM

The original and revised parameterizations for both SWO-ORGANON and the SMCORGANON were chosen to illustrate the expected signs on the parameters and magnitude of those parameters upon Equation (4.1).

### 4.5.1 SWO-ORGANON Parameters

The parameter estimates for the original SWO-ORGANON five-year PM equation and the revised SWO-ORGANON five-year PM equation are shown in Table 4.3.

Table 4.3. The parameter estimates for the original SWO-ORGANON five-year PM equation and the revised SWO-ORGANON five-year PM equation.

| Parameter | Original SWO-ORGANON | Revised SWO-ORGANON |
| :---: | ---: | ---: |
| $a_{0}$ | -0.149558 | -4.64848327 |
| $a_{1}$ | -0.203923 | -0.26655869 |
| $a_{2}$ | 0.0 | 0.00369911 |
| $a_{3}$ | -7.32001 | -2.11802664 |
| $a_{4}$ | 0.00168508 | 0.00336134 |
| $a_{5}$ | 0.0133533 | 0.02549943 |
| $a_{6}$ | 0.0 | 0.01355395 |
| $a_{7}$ | 0.0 | -2.72347095 |

A comparison of the signs on the parameter estimates in Table 4.3 shows that, for both equations, predicted PM will decrease as CR increases, and that predicted PM will increase as both SI and BAL increases. For the original SWO-ORGANON equation, the negative sign on $\mathrm{a}_{1}$ and a zero value of $\mathrm{a}_{2}$ indicates that predicted PM will decrease as D increases. For the revised SWO-ORGANON equation, the negative sign on $\mathrm{a}_{1}$ and a positive sign on $\mathrm{a}_{2}$ indicates that predicted PM will first decrease and then increase as D increases. Therefore, all of the parameters for both equations meet expectations, which is the first requirement for reasonable behavior of the PM predictions from Equation (4.1).

### 4.5.2 SMC-ORGANON Parameters

The parameter estimates for the original SMC-ORGANON five-year PM equation and the revised SMC-ORGANON five-year PM equation are shown in Table 4.4.

Table 4.4. The parameter estimates for the original SMC-ORGANON five-year PM equation and the revised SMC-ORGANON five-year PM equation.

| Parameter | Original SMC-ORGANON | Revised SMC-ORGANON |
| :---: | ---: | ---: |
| $b_{0}$ | -3.27180 | -3.12161659 |
| $b_{1}$ | -0.381656 | -0.44724396 |
| $b_{2}$ | -2.98006 | 0.0 |
| $b_{3}$ | 0.0 | -2.48387172 |
| $b_{4}$ | 0.0112023 | 0.01353918 |
| $b_{5}$ | 0.0182393 | 0.01843137 |

A comparison of the signs on the parameter estimates in Table 4.4 shows that, for both equations, predicted PM will decrease as D and either PCR or XCR increases, and that predicted PM will increase as both SI and BAL increases. Therefore, all of the parameters for both equations meet expectations for young stands, which is the first requirement for reasonable behavior of the PM predictions from Equation (4.1).

### 4.6 Visually Examining the Predictions of PM from the Equations

The PM equations used in ORGANON are functions of several tree and stand attributes and, therefore, the behavior of predicted PM is best examined graphically. I evaluated four Douglasfir PM equations to compare their behavior: the original and revised SWO-ORGANON PM equation and the original and revised SMC-ORGANON PM equation. This analysis was conducted by graphing the predicted PM values across selected values of D, XCR, BAL, and SI for both the original and revised equations on the same graphs. Because the PM equations have four dimensions, it was necessary to use multiple plots to fully explore the predicted behavior of the four PM equations. In reviewing these graphs and the equations behind them, it should be remembered that, while the equations meet biological expectations, there are portions of their multidimensional prediction surface in which trees would not occupy.

### 4.6.1 SWO-ORGANON

Figure 4.1 shows predicted PM graphed for the two SWO-ORGANON equations across D for SI values fixed at 80,120 , and 160 feet and with CR set to one and BAL set to zero. Therefore, these graphs are simulations of the predicted PM for basically open grown trees and, as a result, they should represent the lowest possible values of predicted PM for Douglas-fir trees.

Figure 4.1. Predicted Douglas-fir PM for the original and revised SWO-ORGANON PM models plotted over D for three SI values (e.g., 80, 120, and 160 feet) and with CR fixed to 1.0 and BAL fixed to 0.0 .


Figure 4.1 shows that the predicted PM rates for the original and revised editions of SWOORGANON dramatically vary from each other. These variances are the result of differences in both the modeling data sets and the procedures used to estimate the parameters. The original SWO-ORGANON PM equation for Douglas-fir was restricted to stands with trees primarily between six and 120 years of age, and the parameters of the PM equation was determined by the program RISK. The revised SWO-ORGANON PM equation for Douglas-fir used data that were extended to stands with trees over 400 years of age, and the parameters of the PM equation was determined by the maximum likelihood estimation procedures of SAS.

The result for the revised edition of SWO-ORGANON is the beginning of the expected Ushaped behavior over increasing values of $D$ from a data set that covers a wide range in tree ages. On the other hand, the original edition of SWO-ORGANON equation exhibits the reverse $J$-shaped behavior over increasing values of $D$ that would be expected from young tree data. Both equations exhibit the expected behavior of PM increasing with increasing values of SI.

The reason that the graph of the original SWO-ORGANON equation in Figure 4.1 is so much lower than the revised equation is the result of the $a_{3}$ parameter on CR being nearly $3-1 / 2$ times smaller for the original equation when compared to the revised equation (-7.3 for the original equation and -2.1 for the revised equation, Table 4.3).

The next four graphs show predicted PM from the original and revised SWO-ORGANON equations graphed across CR for BAL values of 0,50, and 100, with OG fixed to 0.33 (its mean value), SI fixed to 120 feet, and D set to 2" in Figure 4.2, 12" in Figure 4.3, 22" in Figure 4.4, and 62" in Figure 4.5.

Figure 4.2. Predicted Douglas-fir PM equations for the original and revised editions of SWOORGANON plotted over CR for BAL values of 0,50 , and 100 square feet per acre with D fixed at 2 ", OG fixed at 0.33 , and a SI fixed at 120 feet.


Figure 4.3. Predicted Douglas-fir PM equations for the original and revised editions of SWOORGANON plotted over CR for BAL values of 0,50 , and 100 square feet per acre with D fixed at $12^{\prime \prime}$, OG fixed at 0.33 , and a SI fixed at 120 feet.

DBH = 12 Inches, $\mathbf{S I}=120$ Feet, and $O G=0.33$


Figure 4.4. Predicted Douglas-fir PM equations for the original and revised editions of SWOORGANON plotted over CR for BAL values of 0,50 , and 100 square feet per acre with D fixed at $22^{\prime \prime}$, OG fixed at 0.33 , and a SI fixed at 120 feet.

DBH = $\mathbf{2 2}$ Inches, $\mathbf{S I}=\mathbf{1 2 0}$ Feet, and $\mathrm{OG}=\mathbf{0 . 3 3}$


Figure 4.5. Predicted Douglas-fir PM equations for the original and revised editions of SWOORGANON plotted over CR for BAL values of 0,50 , and 100 square feet per acre with D fixed at $62^{\prime \prime}$, OG fixed at 0.33 , and a SI fixed at 120 feet.

DBH = 62 Inches, $\mathrm{SI}=120$ Feet, and OG = 0.33


Figures 4.2 through 4.5 all show the expected behaviors of PM decreasing with both increasing CR and decreasing BAL. The original SWO-ORGANON PM equation predicts substantially larger PM for small values of CR than the revised SWO-ORGANON PM equation, with the differences decreasing until the original SWO-ORGANON PM equation crosses the revised equation for $C R$ values roughly between 0.4 to 0.6 , depending upon the value of $D$ and BAL. The large predicted values of PM for the original equation when CR is small compensates for the equation's low predicted values of $P M$ when $C R=1.0$ and $B A L=0.0$ (Figure 4.1). It is possible that the RISK program used in the original analysis stopped at a local minimum that was inferior to the one found by the maximum likelihood method used in
the revised analysis, an outcome that would be made more probable if there were strong correlations between the predictor variables in the Z-functions used in Equation 4.1. For example, Bonate (1999) found that biased parameter estimates can result in a mixed effects model if the correlation between two predictor variables was greater than $|0.5|$. Examination of the SWO PM modeling data set found that the strongest correlations were between D and BAL ( -0.2557 ) and between CR and BAL ( -0.4316 ), values which might have caused problems with RISK finding the optimal parameter values.

The impact of BAL is greater for the revised SWO-ORGANON PM equation than the original, as demonstrated by the wider range in responses for the revised equation. For $\mathrm{D}=62$ " (Figure 4.5), the revised SWO-ORGANON PM equation always predicts higher values of PM than the original ORGANON PM equation due to the inclusion of the $D^{2}$ predictor variable in the revised equation.

The revised SWO-ORGANON PM predictions in Figures 4.2 through 4.5 use an OG value of 0.33 (the average value in the modeling data set). The impact of varying OG upon the revised equation can be assessed by examining the BAL contribution to the logistic link function Z. 2 [i.e., the values calculated from $a_{4} \times B A L+a_{6} \times B A L \times E X P\left(a_{7} \times O G\right)$ for given values of OG]. Figure 4.6 shows the revised SWO-ORGANON BAL contribution to the logistic link function Z. 2 plotted across BAL for four values of OG: 0.003, $0.330,0.763$, and 0.1523 , the smallest, average, midrange, and largest values found in the modeling data set.

Figure 4.6. The revised SWO-ORGANON BAL contribution to the logistic link function Z.2 [i.e., the values calculated from $\mathrm{a}_{4} \times B A L+\mathrm{a}_{6} \times B A L \times E X P\left(a_{7} \times O G\right)$ for given values of $\left.O G\right]$ plotted across BAL for four values of OG (the smallest, mean, midrange, and largest values found in the modeling data set).


The results found in Figure 4.6 show that the BAL contribution to the logistic link function Z. 2 is greatest in stands with small trees, as indicated by OG, with the impact of the BAL contribution decreasing as OG increases. The inclusion of the relationship of the BAL contribution for the midrange value of OG was included to show the negative exponential effect of OG upon the BAL contribution, which is illustrated in Figure 4.7 for a BAL contribution of 400 square feet per acre (the same relative relationship holds for any value of BAL).

Figure 4.7. Contribution of 400 square feet per acre in BAL to the logistic link function Z. 2 plotted across values of OG.


### 4.6.2 SMC-ORGANON

Figure 4.8 shows predicted PM graphed for the two SMC-ORGANON equations across $D$ for SI values of 80,120 , and 160 feet and with CR set to one and BAL set to zero. Therefore, these graphs are simulations of the predicted PM for basically open grown trees and, as a result, they should represent the lowest possible values of predicted PM for Douglas-fir trees.

Figure 4.8. Predicted Douglas-fir PM for the original and revised SMC-ORGANON PM models plotted over D for three SI values (e.g., 80, 120, and 160 feet) and with CR fixed to 1.0 and BAL fixed to 0.0 .


Both of the equations shown in Figure 4.8 exhibit the negative $J$-shape curves expected for relatively young stands of Douglas-fir. Because the data used to model the original and revised SMC-ORGANON PM equations did not support the expected U-shaped equation for larger values of $D$, this behavior will constrict both the type of stands to which the model can be applied, and the duration of the predictions for subsequent stand development. Figure 4.8 also shows that the revised SMC-ORGANON PM equation consistently predicts higher values of PM for simulated open grown trees than the original SMC-ORGANON PM equation. This difference could be the result of adding more modeling data from young stands from the SMC installations to the analysis, and/or it could be the result of changing modeling procedures used to estimate the parameters.

The final three graphs show predicted PM from the original and revised SWO-ORGANON equations graphed across CR for BAL values of 0,50, and 100, with SI fixed to 120 feet, and D set to 2 " in Figure 4.9, 12 " in Figure 4.10, and 22" in Figure 4.11.

Figure 4.9. Predicted Douglas-fir PM equations for the original and revised editions of SMCORGANON plotted over CR for BAL values of 0,50 , and 100 square feet per acre with D fixed at 2" and a SI fixed at 120 feet.


Figure 4.10. Predicted Douglas-fir PM equations for the original and revised editions of SWOORGANON plotted over CR for BAL values of 0,50 , and 100 square feet per acre with D fixed at 12 " and a SI fixed at 120 feet.

DBH = 12 Inches and SI = 120 Feet


Figure 4.11. Predicted Douglas-fir PM equations for the original and revised editions of SWOORGANON plotted over CR for BAL values of 0,50 , and 100 square feet per acre with D fixed at 22 " and a SI fixed at 120 feet.

DBH = 22 Inches and SI = 120 Feet


Examining Figures 9 through 11, one finds that the revised SMC-ORGANON PM equation predicts higher PM rages for 2" trees, followed by lower rates for 12 " and 22 " trees. Both equations meet the expectations that PM will decline with increasing CR and decreasing BAL.

### 4.7 Maximum Size-Density Trajectory

The maximum size-density trajectory is used as an option in ORGANON to restrict stand development in a manner that keeps the stand on or below the maximum size-density trajectory as it develops over time (Hann and Wang 1990, Hann et al. 2003). The following is the maximum size-density trajectory equation used in ORGANON:

$$
\begin{equation*}
\ln \left(\mathrm{QMD}_{\mathrm{i}}\right)=\mathrm{C}_{1}-\mathrm{C}_{2} \cdot \ln \left(\mathrm{~N}_{\mathrm{i}}\right)-\left(\mathrm{c}_{1} \cdot \mathrm{C}_{4}\right) \cdot\left(\mathrm{N}_{\mathrm{i}} / \mathrm{N}_{0}\right)^{\mathrm{c3}} \tag{4.3}
\end{equation*}
$$

Where,

QMD $_{i}=$ The quadratic mean diameter at the $i^{\text {th }}$ measurement $\mathrm{N}_{\mathrm{i}}=$ Number of trees per acre at the $\mathrm{i}^{\text {th }}$ measurement $N_{0}=$ Number of trees per acre just prior the start of competition induced mortality

The parameters $c_{1}$ and $c_{2}$ define the intercept and slope, respectively, of the maximum sizedensity line of:

$$
\ln \left(\mathrm{QMD}_{\mathrm{i}}\right)=\mathrm{C}_{1}-\mathrm{C}_{2} \cdot \ln \left(\mathrm{~N}_{\mathrm{i}}\right)
$$

The remaining part of Equation (4.3) defines the trajectory that a stand follows as it approaches the maximum size-density line.

I have done a lot of work in modeling maximum size-density trajectories using the log of size and the log of density as response and predictor variables, respectively. All of this work has used time series (the longer the measurement series the better) or pooled data sets in which single measurements of size-density are measured on multiple plots. I strongly believe that only repeated measurements should be used to define the maximum size-density line for a species. Sackville Hamilton et al. (1995) supported this belief when they concluded on page 571 that:
"For analysis of thinning trajectories where an initial vertical slope gradually changes towards an asymptote of $-3 / 2 \ldots$, ideally a curve should be fitted to all points in the data set, including the approach to the limiting line. This would provide an objective estimate of the asymptotic slope and the potential for accurately estimating its confidence limits." Ignorant of all of my previous work in the forestry literature (described below), Sackville Hamilton et al. (1995) further concluded that a study taking this approach "...is a rare exception."

I was the minor professor on Nick Smith's PhD work. He had planted red alder seeds in pots using three replicates of three densities and measured their average biomass per tree and surviving number of stems per unit area over time. He needed a way to objectively analyze the data. At that time, all previous maximum size-density (or maximum density-size) studies had either subjectively hand drawn a line above the data cloud or had subjectively selected a subset of the data believed to be at maximum size-density and then fit a line to the data using linear least squares regression or principal component analysis. I decided to characterize how the average tree size asymptotically approached the maximum size-density line as mortality reduced the number of trees on the plot (or pot). The resulting model form predicts the log of average tree size as a function of the log of initial number of trees per unit area (planting density in plantations) and the log of number of surviving trees per unit area. The fixed effects nonlinear regression method was used to estimate the parameters of the resulting maximumsize density line and its trajectory. We also used a published data set for high site index red pine planted at six planting densities at the Petawawa Forest, Ontario, Canada to illustrate the application of the maximum size-density trajectory to more mature stands using average volume per tree as the measure of average tree size.

This, therefore, was the first truly objective method developed for evaluating maximum sizedensity relationships and it would prove to be very powerful for statistically testing alternative hypotheses. The first hypothesis tested was whether or not the slope of the maximum sizedensity line was significantly different from the hypothesized value of $-3 / 2$ for the two data sets. The slopes for both species proved to be not significantly different from the hypothesized value. The second hypothesis tested was whether or not the Suchatschew effect (Harper 1977) was valid. Under this hypothesis, productivity affects the rate of mortality (i.e., how fast the stand moves along the trajectory) but not the shape of the trajectory. Nick had planted his red alder seedlings in two types of soil with different levels of productivity. I designed the appropriate modifications to the maximum size-density trajectory for testing this hypothesis. The results of the statistical test confirmed the validity of the Suchatschew effect.

We submitted this work (both red alder and red pine) to the Canadian Journal of Forest Research where it received accelerated editing because the Associate Editor thought it to be highly significant (the only paper in my career to receive this treatment). The Smith and Hann (1984) article was the result of this work.

While the maximum size-density trajectory equation is static, I pointed out to Nick that the addition of a nonlinear stand level mortality equation to the maximum size-density trajectory equation would result in a dynamic growth model. This we did and the results were published in Smith and Hann (1986).

Hann and Wang (1990) used pooled data from 93 plots to characterize the average population boundary line and average trajectory line of Smith and Hann (1984) for Douglas-fir in southwest Oregon. Because of the limited nature of the time series data, they fixed the slope parameter of the maximum size-density relationship to -0.62305. They then demonstrated how the resulting maximum size-density trajectory could be incorporated into SWO-ORGANON's tree-level mortality equation to guarantee reasonable behavior on long projections. If the uncorrected tree mortality equations placed the stand above the maximum size-density trajectory, then a correction factor was determined for the tree-level mortality equation that increased mortality enough to place the stand on the maximum size-density trajectory.

I was also the minor professor for Klaus Puettmann who was studying the development of red alder and Douglas-fir in pure stands of each and in mixture for his PhD. Again, I designed the model forms and statistical analyses to test the various hypotheses he was interested in examining (again demonstrating the power of the Smith and Hann 1984 approach for modeling the maximum size-density trajectory). Using quadratic diameter as the measure of average size, the following hypotheses were tested for pure red alder stands:

1. The intercept of the maximum size-density line is inversely related to initial density.
2. The curvature of the maximum size-density trajectory varies by initial density.
3. The start of density dependent mortality occurs at different relative densities that vary by initial density.
4. The maximum size-density trajectory differs between natural stands and plantations. The fifth and final hypothesis examined whether the maximum size-density trajectory of red alder different from that of Douglas-fir and, if it did, in what manner did the trajectories differ.

Results of this study were:

1. The intercept of the maximum size-density line is not inversely related to initial density.
2. The curvature of the maximum size-density trajectory does not vary by initial density.
3. The start of density dependent mortality occurs at the same relative density that does not vary by initial density and is parallel to the maximum size-density line.
4. The maximum size-density trajectory does not differ between natural stands and plantations.
Finally, it was found that the maximum size-density trajectories differed between species, with Douglas-fir having a higher intercept and a shallower negative slope for the maximum sizedensity line, and red alder having a trajectory that took a longer range of density to approach the maximum size-density line than Douglas-fir. These analyses were made using pooled data sets of 24 red alder plots and 58 Douglas-fir plots and the fixed effects nonlinear regression method. The results led to the Puettmann et al. (1993) article.

Puettmann et al. (1992) extended the maximum size-density trajectory to species mixture using pooled time series data from 38 pure red alder plots, 58 pure Douglas-fir plots, and 9 mixed plots. I again developed the model forms needed to do this analysis and the fixed effects nonlinear regression method was used to estimate the parameters and associated statistics. The result was a maximum size-density surface rather than line. As with Smith and Hann (1986), we also developed a mixed species mortality model in order to examine the growth dynamics of mixed species stands as they develop over time. As of 2014, this is the only maximum size-density work in mixed species stands that has modeled the interaction between species, and this is the only study that has objectively described the maximum sizedensity surface for a species mixture.

Hann et al. (2003) used the fixed effects linear regression method to estimate the parameters for each of 128 Douglas-fir control plots and 39 western hemlock control plots to test how often the reciprocal of Reineke's (1933) slope parameter characterized the maximum size-density slopes of the various plots. They found that the reciprocal of Reineke's (1933) slope parameter could not be rejected with $\mathrm{P}=0.01$ on 101 of the Douglas-fir plots and 35 of the western hemlock plots. The intercept values of these 101 Douglas-fir plots were quite variable in size indicating that Douglas-fir has multiple maximum size-density lines. These various intercepts were therefore used to explore whether the different maximum size-density lines could be predicted from site index, percent of the basal area in Douglas-fir, or stand origin. However, none of these proved useful for predicting the intercept terms. Density related differences in potential yield have been found in numerous other tree species, including loblolly pine (DeBell et al, 1989, Hasenauer et al. 1994). Because of these findings, the ORGANON model was modified in order to allow the user to specify the intercept of the maximum size-density line.

Hann et al. (2003) also modified the maximum size-density trajectory for the common situation in which initial density and associated quadratic diameter are not known. This modified model form was employed to parameterize the full maximum-size density line and trajectory using quadratic mean diameter as the measure of size and a pooled data set composed of 26 plots from eight long term Douglas-fir installation plots in which the data was judged as covering most of the full trajectory. The fixed effects nonlinear regression method was used to estimate the parameters. The results indicated that:

1. The intercepts did differ across the eight installations. However, three of the installations had intercept values that were not statistically significant from each other and therefore they were combined together into one population boundary line. As a result, this part of the study found six distinct population boundary lines.
2. Reineke's (1933) slope of -0.62305 could not be rejected for the eight installations.

These results agreed with the analyses reported in Hann et al. (2003) and examined earlier in this paper for modeling just the maximum size-density line.

Hann et al. (2003) also improved upon the methodology of Hann and Wang (1990) that incorporated the maximum size-density trajectory into the ORGANON tree-level morality models.

Finally, Hann et al. (2003) also selected for investigation all of the permanent plot data from Douglas-fir fertilization installations that were ocularly judged to be on the maximum sizedensity line. This resulted in pooled data from 86 control plots and 148 fertilized plots on 43 fertilization installations for statistically testing whether or not the intercept of the maximum size-density line was affected by fertilization using fixed effects linear regression methods. The results indicated that fertilization did not affect the intercept of the maximum size-density line further confirming the previous findings of Smith and Hann (1984) that the Suchatschew effect was valid.

The need to include the usage of the maximum size-density trajectory in ORGANON as a user available option arose out of the concern that the relatively limited PM modeling data available at the time would produce PM models that could drive predicted stand development to unreasonable values. The more recent work of CIPS using greatly expanded PM modeling data sets has found that the resulting increment and PM equations now project stand development that result in estimated maximum size-density trajectories that are reasonable without the need to use the limit on maximum size-density option.

To demonstrate, Figures 4.12 through 4.14 show the results of projecting the six plots from one of the SMC Type III spacing studies to an age of 80 years since seed using an early edition of CIPSANON. Figure 4.12 presents the development of Stand Density Index (SDI) over stand age for initial planting densities of 100, 200, 300, 450, 700, and 1200 trees per acre (TPA).

Figure 4.12. Development of SDI over the number of years since seed for initial planting densities of 100, 200, 300, 450, 700, and 1200 TPA.


If the stands are approaching a common maximum size-density line that has a Reineke (1933) slope of -0.62305 , then their predictions should all be plateauing at a common value of SDI. The ending values of SDI for planting densities of 300 through 1200 TPA ranged from 473.57 to 483.49 with an average of 478.68 , which compares very favorably to the average value of 483 reported by Hann et al. (2003) for the control plots on the LOGS installations. This result supports the concept that the different planting densities projected by CIPSANON are approaching a common maximum SDI of reasonable size.

Figure 4.13 shows the development of the maximum size-density trajectory over time, using QMD as the measure of stand size, for initial planting densities of $100,200,300,450,700$, and 1200 number of trees per acre.

Figure 4.13. Maximum size-density trajectory, using QMD as the measure of stand size, for initial planting densities of $100,200,300,450,700$, and 1200 TPA.


The maximum size-density trajectory predicts that, as average stand size develops over time, the TPA of the stand will begin to decrease at an accelerating rate as the stand approaches the common maximum size-density line. Examination of Figure 4.13 shows that the predicted stand development for the six planting densities appear to be following the expected behavior of the maximum size density trajectory.

If the maximum size-density line is defined by Reineke's (1933) relationship, then the slope of the maximum size-density time should be -0.62305 . Figure 4.14 shows just the maximum size density trajectory of the planting density of 1200 TPA.

Figure 4.14. Maximum size-density trajectory, using QMD as the measure of stand size, for initial planting density of 1200 TPA.


Calculation of the slope for the maximum size-density trajectory for initial planting density of 1200 TPA between ages 50 and 80 years since seed results in a value of -0.68322 , which is $9.7 \%$ larger than that of Reinke's (1933) value.

These results support my belief that good G\&Y models are primarily the result of having good quality modeling data sets and good quality model forms.

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### 5.0 Crown Recession Equation ( $\triangle \mathrm{HCB}$ )

CR is used in the $\Delta \mathrm{D}, \Delta \mathrm{H}, \mathrm{PM}$, the largest crown width, the crown profile, and the volume and taper equations of ORGANON. CR is related to HCB via the relationship of:
$C R=1.0-\mathrm{HCB} / \mathrm{H}$
Therefore, predicting change in CR involves predicting both $\Delta \mathrm{H}$ and $\Delta \mathrm{HCB}$.
$\Delta \mathrm{HCB}$ is predicted either directly using a dynamic equation or indirectly using a static HCB equation (Hann and Hanus 2004, Weiskittel et al. 2011).

Of the four attributes commonly predicted by an individual-tree model (e.g., $\Delta \mathrm{D}, \Delta \mathrm{H}, \mathrm{PM}$, and $\Delta H C B), \Delta H C B$ is the most difficult to estimate accurately. This is partly because data for developing direct predictors of $\triangle H C B$ equations are commonly limited in number, errors in measuring HCB often are high, and $\triangle \mathrm{HCB}$ is inherently stochastic. However, it is expected that significant gains in the accuracy of growth and yield projections could be achieved with inclusion of accurate equations that directly predict $\triangle \mathrm{HCB}$, particularly given that $\triangle \mathrm{HCB}$ is also closely related to silvicultural treatments (Liu et al. 1995).

### 5.1 An Example Equation for Directly Predicting $\triangle H C B$

As of 2004, only a hand full of studies had explored the direct modeling of $\triangle H C B$ for use in a nonspatial model such as ORGANON. Most of these alternatives were compared in Hann and Hanus (2004). The following logistic based model form, and associated parameter estimates, from Hann and Hanus (2004) were inserted into the SMC-ORGANON model and evaluated for the resulting predictive behavior:

$$
\begin{equation*}
\Delta H C B=\frac{C L+P \Delta H}{1.0+e^{a_{0}+a_{1} X_{1}+a_{2} X_{2}+a_{3} X_{3}+a_{4} X_{4}+a_{5} X_{5}}} \tag{5.1}
\end{equation*}
$$

Where,

$$
\begin{aligned}
& X_{1}=\ln (C R) \\
& X_{2}=C R \\
& X_{3}=G E A \\
& X_{4}=\ln (C C F+1.0) \\
& X_{5}=C R / C C F
\end{aligned}
$$

Equation (5.1) limits the value of predicted $\Delta \mathrm{HCB}$ to fall between zero and $\mathrm{CL}+\Delta \mathrm{H}$ (which is the maximum CL at the end of the growth period). The equation was fit using the same data set that was used to develop the $\Delta \mathrm{H}$ equation for the revised SMC-ORGANON model
discussed in Chapter 3. The parameter estimates for Equation (5.1) from Hann and Hanus (2004) are found in Table 5.1.

Table 5.1. The parameter estimates for Equation (5.1) from Hann and Hanus (2004).

| Parameter | Parameter Estimate |
| :---: | ---: |
| $\mathrm{a}_{0}$ | -5.4068 |
| $\mathrm{a}_{1}$ | -4.1626 |
| $\mathrm{a}_{2}$ | 6.4714 |
| $\mathrm{a}_{3}$ | 0.0566 |
| $\mathrm{a}_{4}$ | -0.2320 |
| $\mathrm{a}_{5}$ | 128.0030 |

Graphs of predicted relative crown recession $[\Delta \mathrm{HCB} /(\mathrm{CL}+\Delta \mathrm{H})]$ from the Equation (5.1) are found in Figures 5.1 to 5.4. All four graphs show relative crown recession plotted across CR for CCF values of $10,210,410$, and $610 \%$. GEA is fixed to $5,25,45$, and 65 years for Tables 5.1, $5.2,5.3$, and 5.4 , respectively.

Figure 5.1 Relative crown recession predicted by Equation (5.1) plotted across the tree's measured CR, the plot's measured CCF, and a GEA of 5 years.

Predicted Relative Crown Recession at GEA = 5


Figure 5.2 Relative crown recession predicted by Equation (5.1) plotted across the tree's measured CR, the plot's measured CCF, and a GEA of 25 years.

Predicted Relative Crown Recession at GEA=25


Figure 5.3 Relative crown recession predicted by Equation (5.1) plotted across the tree's measured CR, the plot's measured CCF, and a GEA of 45 years.

Predicted Relative Crown Recession at GEA=45


Figure 5.4 Relative crown recession predicted by Equation (5.1) plotted across the tree's measured CR, the plot's measured CCF, and a GEA of 65 years.

Predicted Relative Crown Recession at GEA=65


An evaluation of Equation (5.1) was conducted by taking several young plantations and projecting their development over the proceeding 80-years of development. The same young plantations were also projected using the traditional, indirect method of predicting $\triangle \mathrm{HCB}$ discussed in the next sections of this Chapter. The results showed that the direct method produced estimates of BA and total stem cubic foot volume per acre that were much higher than that produced by the indirect method of predicting $\triangle \mathrm{HCB}$. Examination of the predicted values of CR for the trees on these plots showed that the direct method produced much longer crowns at the end of the projections than the indirect method. Examination of Figures 5.1 to 5.4 show that predicted $\Delta H C B$ from Equation (5.1) decreases to small values at 45 and 65 years of development, which is causing these longer, unreasonable crown lengths.

The data set used to fit Equation 5.1 had an average breast height age of 20.8 years and a maximum breast height age of 50 years. It is suspected that the basic behavior of $\triangle \mathrm{HCB}$ changes over a longer period of time than that found for $\Delta \mathrm{D}$ or $\Delta \mathrm{H}$ (both of which peak at young ages). As a result, it is not possible to develop a universally applicable $\Delta \mathrm{HCB}$ model using data from just young stands available to develop Equation (5.1). These disappointing findings resulted in the continued usage of the indirect method for predicting $\triangle \mathrm{HCB}$ in SMCORGANON (and all other versions of ORGANON).

### 5.2 Basic Equation for Indirectly Predicting $\triangle$ HCB

The following model form has been used in all four versions of ORGANON to predict $\triangle H C B$ indirectly from a static HCB equation for trees growing in untreated stands:
$\Delta \mathrm{HCB}=\mathrm{PHCB}_{2}-\mathrm{PHCB}_{1}$
Where,
PHCB $_{i}=$ Predicted HCB at either the start, $i=1$, or the end, $i=2$, of the five-year growth period
$\mathrm{PHCB}_{\mathrm{i}}=\left\{\mathrm{H}_{\mathrm{i}}-\mathrm{K}_{2}\right\} \times\left\{1+\mathrm{EXP}\left[\mathrm{b}_{0}+\mathrm{b}_{1} \mathrm{H}_{\mathrm{i}}+\mathrm{b}_{2} \mathrm{CCFL}_{i}+\mathrm{b}_{3} \ln \left(\mathrm{BA}_{\mathrm{i}}\right)+\mathrm{b}_{4}\left(\mathrm{D}_{\mathrm{i}} / \mathrm{H}_{\mathrm{i}}\right)+\mathrm{b}_{5}\left(\mathrm{SI}-\mathrm{K}_{1}\right)+\mathrm{b}_{6} \mathrm{OG}_{\mathrm{i}}^{2}\right]\right\}^{-1}+\mathrm{K}_{2}$
$H_{i}=H$ at either the start, $i=1$, or the end, $i=2$, of the five-year growth period
$C C F L_{i}=C C F L$ at either the start, $i=1$, or the end, $i=2$, of the five-year growth period
$B A_{i}=B A$ at either the start, $i=1$, or the end, $i=2$, of the five-year growth period
$D_{i}=D$ at either the start, $i=1$, or the end, $i=2$, of the five-year growth period
$\mathrm{SI}=\mathrm{SI}_{\text {HS }}$ for SWO-ORGANON and $\mathrm{SI}_{\mathrm{B}}$ for SMC-ORGANON
$O G_{i}=$ An old growth indicator at either the start, $\mathrm{i}=1$, or the end, $\mathrm{i}=2$, of the five-year growth period
$\mathrm{OG}_{\mathrm{i}}=\mathrm{D} 5_{\mathrm{i}} \times \mathrm{H} 5_{\mathrm{i}} / 10,000$
$D 5_{i}=$ The average $D$ of the five largest diameter trees per acre in Douglas-fir, white fir, grand fir, ponderosa pine, sugar pine, and/or incense-cedar at either the start, $\mathrm{i}=1$, or the end, $\mathrm{i}=2$, of the five-year growth period
$H 5{ }_{i}=$ The average $H$ of the five largest diameter trees per acre in Douglas-fir, white fir, grand fir, ponderosa pine, sugar pine, and/or incense-cedar at either the start, $\mathrm{i}=1$,
or the end, $\mathrm{i}=2$, of the five-year growth period
The term $b_{0}+b_{1} H_{i}+b_{2} C C F L_{i}+b_{3} \ln \left(B A_{i}\right)+b_{4}\left(D_{i} / H_{i}\right)+b_{5}\left(S I-K_{1}\right)+b_{6} O_{i}^{2}$ is the logistic link functions in Equation (5.3). The usage of $\mathrm{H}_{2}, \mathrm{CCFL}_{2}, \mathrm{BA}_{2}, \mathrm{D}_{2}$ and $\mathrm{OG}_{2}$ in Equation (5.3) requires that $\Delta \mathrm{D}$, $\Delta \mathrm{H}$, and PM be calculated before Equation (5.2) can be calculated. If predicted $\Delta \mathrm{HCB}$ from Equation (5.2) is less than zero, then it is set to zero.

### 5.3 Rationale for the Form of Equation (5.3), and the Presence of the $K_{1}$ and $K_{2}$ Constants

The basic HCB equation form applied in Equation (5.3) is a modified logistic model form that has been used to predict HCB in SWO-ORGANON (Ritchie and Hann 1987 and Hanus et al. 2000), NWO-ORGANON (Zumrawi and Hann 1989), SMC-ORGANON (Hann et al. 2003), and RAP-ORGANON (Hann et al. 2011). Equation (5.3) constrains HCB to values between $\mathrm{K}_{2}$ and H. It is expected that HCB will increase with an increase in tree size (e.g., H), with an increase in the level of one sided competition (e.g., CCFL), and with an increase in the level of two sided competition (e.g., BA). It is further expected that HCB will decrease with a decrease in tree form (e.g., D/H), and with an increase in the level of productivity (e.g., SI). This means that $b_{1}, b_{2}$, and $b_{3}$ should have negative signs and that $b_{4}$ and $b_{5}$ should have positive signs.

Hann et al. (1987) noted that the Europeans have used $H / D$ as a measure of tree form, and Walters and Hann (1986) used H/D in their taper equations and in their equation for predicting CR . One problem with the usage of H/D is that it approaches infinity as D approaches zero. This problem is resolved when $\mathrm{D} / \mathrm{H}$ is used instead. There is also a trend of $\mathrm{D} / \mathrm{H}$ declining with position in the stand, with suppressed trees having smaller values of D/H than dominant trees.
$\mathrm{K}_{1}$ was set to 0.0 for the original edition of SWO-ORGANON and for both editions of SMCORGANON but it was mistakenly set to 4.5 in the revised edition of SWO-ORGANON (in effect, it subtracts $b_{5} \times 4.5$ from $b_{0}$ ).

The $\mathrm{K}_{2}$ constant was added to the basic HCB equation because analysis of the red alder plantation data indicated that HCB did not approach zero when BA approached zero (Hann et al. 2011). The resulting value of $\mathrm{K}_{2}$ was 2.0 -feet for red alder growing in plantations. The value of $\mathrm{K}_{2}$ was zero prior to the development of RAP-ORGANON.

### 5.4 Statistical Methods for Estimating the Common Parameters of Equation (5.3)

The common parameters of Equations (5.3) and (5.4) are fit using the following transformed model form:
$A B R=\left\{1+E X P\left[b_{0}+b_{1} H_{i}+b_{2} C_{C F L}^{i}+b_{3} \ln \left(\mathrm{BA}_{\mathrm{i}}\right)+\mathrm{b}_{4}\left(\mathrm{D}_{\mathrm{i}} / \mathrm{H}_{\mathrm{i}}\right)+\mathrm{b}_{5}\left(\mathrm{SI}^{-K_{1}}\right)+\mathrm{b}_{6} \mathrm{OG}_{\mathrm{i}}{ }^{2}\right]\right\}^{-1}$
Where,

$$
\text { ABR = Adjusted Bole Ratio, }\left(\mathrm{HCB}-\mathrm{K}_{2}\right) /\left(\mathrm{H}-\mathrm{K}_{2}\right)
$$

When $\mathrm{K}_{2}=0.0$, then the response variable in Equation (5.4) becomes HCB/H, which is the definition of bole ratio. Transformed Equation (5.4) is used because it usually homogenizes the variance about Equation (5.3). The parameter estimates of Equation (5.4) are determined using unweighted, nonlinear regression.

### 5.5 Description of the HCB Data Sets Used in the Following Examples

The initial versions of SWO-ORGANON and SMC-ORGANON estimated the parameters of Equation (5.3), with $\mathrm{K}_{2}$ set to zero, and using all trees with measured values of HCB. Hanus et al. (2000) found that various types of damage caused significant changes to HCB. Therefore, in the revised versions of SWO-ORGANON and SMC-ORGNON, the HCB data used to model Equation (5.3) were restricted to undamaged trees under the assumption that those trees would provide the most accurate estimate of $\Delta \mathrm{HCB}$ using the indirect method of Equation (5.2).

### 5.5.1 SWO-ORGANON Data Sets

The original modeling data set contained 391 plots and, of these, 126 plots were eliminated because they had been thinned within the past 20 years. This was done to ensure that the crowns had stabilized after cutting. Of the remaining plots, 237 were used for modeling and the remaining 28 plots were used for validation. The data description in Ritchie and Hann (1987) was limited to D and H , and they included both damaged and undamaged trees in their modeling data set.

The revised data set contained a combination of the 529 plots, with 526 plots containing at least one Douglas-fir that was above breast height. However, only 407 of the plots in the combined data set that contained Douglas-fir were not affected by cutting. In addition, the change in the sampling design for the revised data set resulted in some large diameter trees being removed from the original data set. These changes removed a total of 3,028 Douglas-fir trees from the original modeling data set. Only undamaged trees were used to estimate the parameters of Equation (5.3) using its transformed Equation (5.4). A summary of the descriptive statistics for the original and revised Douglas-fir modeling data sets is found in Table 5.2.

Table 5.2. Descriptive statistics for the Douglas-fir data set used to model HCB in the original SWO-ORGANON analysis and the revised SWO-ORGANON analysis, D and D5 were measured in inches, $\mathrm{H}, \mathrm{H} 5$, and $\mathrm{SI}_{\mathrm{HS}}$ were measured in feet, CCFL was calculated in percent, and BA was calculated in square feet per acre.

|  | Original Analysis $(\mathrm{N}=9778)$ |  |  | Revised Analysis $(\mathrm{N}=8236)$ |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Attribute | Mean | Minimum | Maximum | Mean | Minimum | Maximum |
| D | 12.8 | 0.1 | 84.0 | 14.4 | 0.1 | 81.3 |
| H | 74.4 | 4.6 | 210.3 | 83.9 | 4.6 | 244.2 |
| CCFL | NA | NA | NA | 104.1 | 0.0 | 485.6 |
| BA | NA | NA | NA | 206.9 | 1.4 | 440.0 |
| SI | NA | NA | NA | 99.4 | 41.5 | 142.7 |
| D5 | NA | NA | NA | 27.4 | 2.1 | 67.1 |
| H5 | NA | NA | NA | 110.0 | 13.0 | 230.2 |

### 5.5.2 SMC-ORGANON Data Sets

The original modeling data set contained "untreated" data from 686 plots consisting of all damaged and undamaged trees with actual HCB measurements from (1) untreated control plots, (2) plots that had been just thinned and for which the CCFL and BA values just before thinning were known, and (3) all measurements from plots that had been thinned more than 20 -years ago.

The revised edition of SMC-ORGANON used both the original data set and more recent measurements taken on the SMC installations. The modeling data set was restricted to undamaged trees with measurements of HCB at the start and end of the growth period (Hann and Hanus 2004). A comparison of the original and revised data sets is found in Table 5.3.

Table 5.3. Descriptive statistics for the Douglas-fir data set used to model HCB in the original SMC-ORGANON analysis and the revised SMC-ORGANON analysis, D was measured in inches, H and $\mathrm{SI}_{B}$ were measured in feet, CCFL was calculated in percent, and BA was calculated in square feet per acre.

|  | Original Analysis $(\mathrm{N}=11,746)$ |  | Revised Analysis $(\mathrm{N}=5341)$ |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Attribute | Mean | Minimum | Maximum | Mean | Minimum | Maximum |
| D | 9.4 | 0.6 | 46.0 | 8.1 | 1.6 | 25.6 |
| H | 64.9 | 8.0 | 188.0 | 51.1 | 14.1 | 139.0 |
| CCFL | 133.5 | 0.0 | 532.7 | 85.4 | 0.0 | 464.9 |
| BA | 140.8 | 36.6 | 406.1 | 128.9 | 13.1 | 331.0 |
| SI | 118.5 | 77.6 | 155.0 | 119.8 | 77.6 | 162.1 |

### 5.6 Example Parameterizations for $\triangle H C B$

The original and revised parameterizations, and associated K value, for both SWO-ORGANON and the SMC-ORGANON were chosen to illustrate the expected signs on the parameters and magnitude of those parameters upon Equation (5.3).

### 5.6.1 SWO-ORGANON Parameters

The first set of parameters in Table 5.4 was developed for the original edition of SWOORGANON (Ritchie and Hann 1987) and the second set of parameters is those used in the revised edition of SWO-ORGANON (Hanus et al. 2000).

Table 5.4. The parameter estimates of Equation (5.3) for both the original edition of SWOORGANON and the revised edition of SWO-ORGANON.

| Parameter | Original SWO-ORGANON | Revised SWO-ORGANON |
| :---: | ---: | ---: |
| $\mathrm{b}_{0}$ | 2.59959 | 1.797136911 |
| $\mathrm{~b}_{1}$ | -0.00725950 | -0.010188791 |
| $\mathrm{~b}_{2}$ | -0.00458228 | -0.003346230 |
| $\mathrm{~b}_{3}$ | -0.441557 | -0.412217810 |
| $\mathrm{~b}_{4}$ | 1.61311 | 3.958656001 |
| $\mathrm{~b}_{5}$ | 0.00467539 | 0.008526562 |
| $\mathrm{~b}_{6}$ | 0.0 | 0.448909636 |
| $\mathrm{~K}_{1}$ | 0.0 | 4.5 |
| $\mathrm{~K}_{2}$ | 0.0 | 0.0 |

A comparison of the nonlinear regression parameter estimates, $b_{0}$ to $b_{6}$, in Table 5.4 shows that both sets of parameters exhibit the same signs, which is the first requirement for creating reasonable behavior of the HCB predictions from Equation (5.3). The prominent changes between the revised and original parameterizations of Equation (5.3) is the addition of $\mathrm{OG}^{2}$, which affects the size of $b_{0}$, the $145 \%$ increase in $b_{4}$, which affects the size of the impact of $\mathrm{D} / \mathrm{H}$ in the equation, and the $82 \%$ increase in $\mathrm{b}_{5}$, which affects the size of the impact of $\mathrm{SI}_{\mathrm{HS}}$ in the equation.

### 5.6.2 SMC-ORGANON Parameters

The first set of parameters in Table 5.5 was developed for the original edition of SMCORGANON (Hann et al. 2003) and the second set of parameters are those used in the revised edition of SMC-ORGANON (Hann and Hanus. 2004).

Table 5.5. The parameter estimates of Equation (5.3) for both the original edition of SMCORGANON and the revised edition of SMC-ORGANON.

| Parameter | Original SMC-ORGANON | Revised SMC-ORGANON |
| :---: | ---: | ---: |
| $\mathrm{b}_{0}$ | 3.411317351 | 6.18464679 |
| $\mathrm{~b}_{1}$ | -0.009947861 | -0.00328764 |
| $\mathrm{~b}_{2}$ | -0.001906272 | -0.00136555 |
| $\mathrm{~b}_{3}$ | -0.656269205 | -1.19702220 |
| $\mathrm{~b}_{4}$ | 4.520522655 | 3.17028263 |
| $\mathrm{~b}_{5}$ | 0.002595706 | 0.0 |
| $\mathrm{~b}_{6}$ | 0.0 | 0.0 |
| $\mathrm{~K}_{1}$ | 0.0 | 0.0 |
| $\mathrm{~K}_{2}$ | 0.0 | 0.0 |

A comparison of the nonlinear regression parameter estimates, $b_{0}$ to $b_{6}$, in Table 5.5 shows that both sets of parameters exhibit the same signs, which is the first requirement for creating reasonable behavior of the HCB predictions from Equation (5.3). The prominent changes between the revised and original parameterizations of Equation (5.3) is the elimination of $b_{5}$, which affects the impact of $\mathrm{SI}_{\mathrm{B}}$ upon Equation (5.3), the $82 \%$ increase in $\mathrm{b}_{3}$, which affects the size of the impact of BA in the equation, the $81 \%$ increase in $\mathrm{b}_{0}$, which is partially caused by the elimination of $b_{5}$, and the $67 \%$ reduction in $b_{1}$, which affects the size of the impact of $H$ in the equation.

### 5.7 Visually Examining the Predictions of HCB from the Equations

Because of the complexity of Equation (5.3), I find it useful to also graphically examine the behavior of the equation. Two sets of example graphs are presented below in order to show the amount of variability that can occur from fitting Equation (5.3) to different data sets. The first set of graphs is for the SWO-ORGANON HCB equations and the second is for the SMCORGANON HCB equations. In reviewing these graphs and the equations behind them, it should be remembered that, while the equations meet biological expectations, there are portions of their multidimensional prediction surface in which trees would not occupy.

### 5.7.1 Original SWO-ORGANON $\triangle H C B$ Equation versus Revised SWO-ORGANON $\triangle H C B$ Equation

Two comparisons are made in order to examine the impact of the alternative data sets upon the resulting parameters and their predictions: the original SWO-ORGANON HCB data set versus the revised SWO-ORGANON HCB data set. Figure 5.5 shows a graph depicting the predictions of the minimum HCB from Equation (5.3) for simulated open grown trees. This was done by fixing CCFL and BA to zero, D/H to a value of 0.4 (which is a large value of D/H, representing a tree with a large value D for its value of H ), and OG to 0.1 . Bole ratio (BR, defined as HCB/H) was then graphed across H for three values of SI ( 80,120 , and 160 -feet). On this graph were plotted predictions of BR from both the original edition and revised edition SWO-ORGANON in order to aid in making comparisons.

Figure 5.5. Predicted Douglas-fir BR for the original and revised SWO-ORGANON HCB equations plotted over H for three SI values (e.g., 80, 120, and 160 feet) and with CCFL fixed to 0.0 , BA fixed to $0.0, \mathrm{D} / \mathrm{H}$ fixed to 0.4 , and OG fixed to 0.1 . The fixed values were chosen to simulate the BR of an open grown tree.

$$
\text { CCFL }=0 \%, B A=0 \text { Sq.Ft./Ac., D/H = } 0.4 \mathrm{In} . / F t ., O G=0.1
$$



The effect of the larger value of the $b_{5}$ parameter for the revised SWO-ORGANON is shown as a wider spread between the three values of SI than that found for the original edition of SWOORGANON. The faster increase in predicted bole ratio as H increases is due to the larger size of the $b_{1}$ parameter for the revised SWO-ORGANON equation than the original equation. However, examination of the scale of the Y -axis shows that BR for a simulated open grown tree is predicted to be very close to zero as SI and H increase, which would be expected for a species that is not self-pruning.

Four additional graphs were prepared to further explore the predictive behavior of SWOORGANON's HCB equations. Each of the four graphs plotted BR across CCFL for three levels of BA (50, 150, and 250 square feet per acre), with SI fixed at 120-feet and OG fixed at 0.1. What differed between the four plots were the values of $H$ and $D / H$, with separate graphs for $H=35$-feet and $\mathrm{D} / \mathrm{H}=0.05$ or 0.25 and for $\mathrm{H}=135$-feet and $\mathrm{D} / \mathrm{H}=0.05$ or 0.25 . The value of $\mathrm{D} / \mathrm{H}=0.05$ was chosen to represent a more suppressed tree and the value of $\mathrm{D} / \mathrm{H}=0.25$ was chosen to represent a more dominant tree. Again, both editions of a given version were plotted on the same graph to aid in making comparisons. Finally, all four graphs, and the two versions
displayed on each, exhibit the expected behavior of predicted BR increasing with increasing CCFL and increasing BR.

Figure 5.6 shows predicted BR for both editions of SWO-ORGANON HCB equations plotted over CCFL for three BA values (e.g., 50,150 , and 250 square feet/acre) and with H fixed to 35 -feet, D/H fixed to 0.05 , SI fixed to 120 -feet, and OG fixed to 0.1 . The values of H and D/H were chosen to represent a small tree in the understory of a stand older than 13 -years.

Figure 5.6. Predicted Douglas-fir BR for the original and revised SWO-ORGANON HCB equations plotted over CCFL for three BA values (e.g., 50, 150, and 250 square feet/acre) and with H fixed to 35 -feet, $\mathrm{D} / \mathrm{H}$ fixed to 0.05 , SI fixed to 120 -feet, and OG fixed to 0.1 . The values of $H$ and $D / H$ were chosen to represent a small tree in the understory of a stand older than 13years.
$\mathrm{H}=\mathbf{3 5}$-Feet, $\mathrm{D} / \mathrm{H}=0.05 \mathrm{In} . / \mathrm{Ft} ., \mathrm{SI}=\mathbf{1 2 0}-\mathrm{Feet}, \mathrm{OG}=\mathbf{0 . 1}$


The results shown in Figure 5.6 indicate that the two equations predict similar values of HCB for such a tree. It would be expected that suppressed trees growing in a stand with a high value of BA would also exhibit higher values of CCFL, and vice a versa. As a result, the range of their "reasonable" HCB prediction space would shift with the trees' particular values of CCFL and BA.

Figure 5.7 plots predicted BR for both editions of SWO-ORGANON HCB equations over CCFL for three BA values (e.g., 50, 150, and 250 square feet/acre) and with H fixed to $35-\mathrm{feet}$, D/H
fixed to 0.25 , SI fixed to 120 -feet, and OG fixed to 0.1 . The values of H and $\mathrm{D} / \mathrm{H}$ were chosen to represent a small tree in the overstory of an approximately 13 -years old stand based on its SIHs of 120 -feet.

Figure 5.7. Predicted Douglas-fir BR for the original and revised SWO-ORGANON HCB equations plotted over CCFL for three BA values (e.g., 50, 150, and 250 square feet/acre) and with H fixed to 35 -feet, $\mathrm{D} / \mathrm{H}$ fixed to 0.25 , SI fixed to 120 -feet, and OG fixed to 0.1 . The values of H and $\mathrm{D} / \mathrm{H}$ were chosen to represent a small tree in the overstory of an approximately 13yearold stand based on its $\mathrm{SI}_{\mathrm{HS}}$ of 120 -feet.

$$
\mathrm{H}=35-\text { Feet, } \mathrm{D} / \mathrm{H}=0.25 \mathrm{In} . / \mathrm{Ft} ., \mathrm{SI}=120-\text { Feet, } \mathrm{OG}=0.1
$$



Examination Figure 5.7 shows that the revised equation consistently predicts lower values of HCB than the original equation for such a tree. This difference is the result of the large change in the size of the $b_{4}$ parameter between the two equations. It is expected that dominant trees would exhibit low values of CCFL regardless of the value of BA. As a result, the range of their "reasonable" HCB prediction space would occupy the left hand side of the graph.

Figure 5.8 shows predicted BR for both editions of SWO-ORGANON HCB equations plotted over CCFL for three BA values (e.g., 50, 150, and 250 square feet/acre) and with H fixed to 135 -feet, D/H fixed to 0.05 , SI fixed to 120 -feet, and OG fixed to 0.1 . The values of H and D/H were chosen to represent a large tree in the understory of a stand much older than 57 years based on a SI ${ }_{\text {Hs }}$ value of 120 -feet.

Figure 5.8. Predicted Douglas-fir BR for the original and revised SWO-ORGANON HCB equations plotted over CCFL for three BA values (e.g., 50, 150, and 250 square feet/acre) and with H fixed to 135 -feet, $\mathrm{D} / \mathrm{H}$ fixed to 0.05 , SI fixed to 120 -feet, and OG fixed to 0.1 . The values of $H$ and $D / H$ were chosen to represent a large tree in the understory of a stand much older than 57 years based on a Sl Hs value of 120 -feet.


Review of Figure 5.8 shows that the revised edition of SWO-ORGANON predicts higher values HCB for a large, understory tree than the original edition. As with shorter suppressed trees, it would be expected that taller suppressed trees growing in a stand with a high value of BA would also exhibit higher values of CCFL, and vice a versa. As a result, the range of their "reasonable" HCB prediction space would shift with the trees' particular values of CCFL and BA.

Figure 5.9 plots predicted BR for both editions of SWO-ORGANON HCB equations over CCFL for three BA values (e.g., 50, 150, and 250 square feet/acre) and with H fixed to 135 -feet, D/H fixed to 0.25 , SI fixed to 120 -feet, and OG fixed to 0.1 . The values of H and $\mathrm{D} / \mathrm{H}$ were chosen to represent a large tree in the overstory of an approximately 57 -year-old stand based on its SI Hs value of 120 -feet.

Figure 5.9. Predicted Douglas-fir BR for the original and revised SWO-ORGANON HCB equations plotted over CCFL for three BA values (e.g., 50, 150, and 250 square feet/acre) and with H fixed to 135 -feet, $\mathrm{D} / \mathrm{H}$ fixed to 0.25 , SI fixed to 120 -feet, and OG fixed to 0.1 . The values of H and $\mathrm{D} / \mathrm{H}$ were chosen to represent a large tree in the overstory of an approximately 57years old stand based on its $\mathrm{SI}_{\text {Hs }}$ value of 120 -feet.


Figure 5.9 shows that the revised equation consistently predicts somewhat lower values of HCB than the original equation for such a tree, with the difference increasing with an increase in CCFL. It is expected that dominant trees would exhibit low values of CCFL regardless of the value of BA. As a result, the range of their "reasonable" HCB prediction space would occupy the left-hand side of the graph, where the difference between the two editions is the smallest.

In summary, the addition of the older stand data and the hardwood stand data to the original data set has resulted in the revised HCB Equation (5.3) predicting both higher and lower values of $B R$ depending upon the size of the tree and its position within the stand. This added data resulted in a greater range in H, CCFL, BA, D, and OG than found in the original modeling data set and, as a result, has created a HCB equation that is more widely applicable than the original equation.

### 5.7.2 Original SMC-ORGANON $\triangle H C B$ Equation versus Revised SMC-ORGANON $\triangle H C B$ Equation

Figure 5.10 shows a graph depicting the predictions of the minimum HCB from Equation (5.3) for simulated open grown trees. This was done by fixing CCFL and BA to zero, D/H to a value of 0.4 (which is a large value of $\mathrm{D} / \mathrm{H}$, representing a tree with a large value D for its value of H ), and $O G$ to 0.1 . Bole ratio ( BR , defined as $\mathrm{HCB} / \mathrm{H}$ ) was then graphed across H for three values of $\mathrm{SI}(80,120$, and 160 -feet). On this graph were plotted predictions of BR from both the original edition and revised edition SMC-ORGANON in order to aid in making comparisons.

Figure 5.10. Predicted Douglas-fir BR for the original and revised SMC-ORGANON HCB equations plotted over H for three SI values (e.g., 80, 120, and 160 feet) and with CCFL fixed to 0.0 , BA fixed to $0.0, \mathrm{D} / \mathrm{H}$ fixed to 0.4 , and OG fixed to 0.1 . The fixed values were chosen to simulate the $B R$ of an open grown tree.

The effect of the zero value of the $b_{5}$ parameter for the revised SMC-ORGANON is shown as single line near the origin, and the positive value of $b_{5}$ for the original edition of SMCORGANON exhibits the same type of behavior as found for both editions of SWO-ORGANON. The faster increase in predicted bole ratio as H increases is due to the larger size of the $\mathrm{b}_{1}$ parameter for the original edition SMC-ORGANON equation than the original equation (the small size of the $b_{1}$ parameter for the revised edition resulted in its predicted HCB values being to be too small to discern on the graph). As with SWO-ORGANON, examination of the scale of the Y -axis shows that BR for a simulated open grown tree is also predicted to be very close to zero as SI and H increase.

Four additional graphs were prepared to further explore the predictive behavior of SMCORGANON's HCB equations. Each of the four graphs plotted BR across CCFL for three levels of $\mathrm{BA}(50,150$, and 250 square feet per acre), with SI fixed at 120 -feet and OG fixed at 0.1 . What differed between the four plots were the values of H and $\mathrm{D} / \mathrm{H}$, with separate graphs for $\mathrm{H}=35$-feet and $\mathrm{D} / \mathrm{H}=0.05$ or 0.25 and for $\mathrm{H}=135$-feet and $\mathrm{D} / \mathrm{H}=0.05$ or 0.25 . The value of $\mathrm{D} / \mathrm{H}=0.05$ was chosen to represent a more suppressed tree and the value of $\mathrm{D} / \mathrm{H}=0.25$ was chosen to represent a more dominant tree. Again, both editions of a given version were plotted on the same graph to aid in making comparisons. Finally, all four graphs, and the two versions displayed on each, exhibit the expected behavior of predicted BR increasing with increasing CCFL and increasing BR.

Figure 5.11 shows predicted BR for both editions of SMC-ORGANON HCB equations plotted over CCFL for three BA values (e.g., 50, 150, and 250 square feet/acre) and with H fixed to 35 -feet, D/H fixed to 0.05 , SI fixed to 120 -feet, and OG fixed to 0.1 . The values of H and D/H were chosen to represent a small tree in the understory of a stand older than 11 -years based on a $\mathrm{SI}_{\mathrm{B}}$ value of 120 -feet.

Figure 5.11. Predicted Douglas-fir BR for the original and revised SMC-ORGANON HCB equations plotted over CCFL for three BA values (e.g., 50, 150, and 250 square feet/acre) and with H fixed to 35 -feet, $\mathrm{D} / \mathrm{H}$ fixed to 0.05 , SI fixed to 120 -feet, and OG fixed to 0.1 . The values of H and $\mathrm{D} / \mathrm{H}$ were chosen to represent a small tree in the understory of a stand older than 11years based on a $\mathrm{SI}_{\mathrm{B}}$ value of 120 -feet.


Inspection of Figure 5.11 shows that the revised SMC-ORGANON equation predicts higher values of BR than the original edition for BA of 250 square feet per acre, approximately the same values of $B R$ as the original edition for BA of 150 square feet per acre, and lower values of BR than the original edition for BA of 50 square feet per acre. Again, it would be expected that suppressed trees growing in a stand with a high value of BA would also exhibit higher values of CCFL, and vice a versa. As a result, the range of their "reasonable" HCB prediction space would shift with the trees' particular values of CCFL and BA.

Figure 5.12 plots predicted BR for both editions of SMC-ORGANON HCB equations across CCFL for three BA values (e.g., 50,150 , and 250 square feet/acre) and with H fixed to 35 -feet, $\mathrm{D} / \mathrm{H}$ fixed to 0.25 , SI fixed to 120 -feet, and OG fixed to 0.1 . The values of H and $\mathrm{D} / \mathrm{H}$ were chosen to represent a small tree in the overstory of a young stand of approximately 11 -years old base on a $\mathrm{SI}_{\mathrm{B}}$ value of 120 -feet.

Figure 5.12. Predicted Douglas-fir BR for the original and revised SMC-ORGANON HCB equations plotted over CCFL for three BA values (e.g., 50, 150, and 250 square feet/acre) and with H fixed to 35 -feet, $\mathrm{D} / \mathrm{H}$ fixed to 0.25 , SI fixed to 120 -feet, and OG fixed to 0.1 . The values of H and $\mathrm{D} / \mathrm{H}$ were chosen to represent a small tree in the overstory of a young stand of approximately 11 -years old base on a $\mathrm{SI}_{\mathrm{B}}$ value of 120 -feet.


Figure 5.12 shows that the revised equation predictions for a small overstory tree predicts higher values of HCB than the original equation for BA values of 150 and 250-square feet per
acre and lower values of HCB than the original equation for BA values of 50-square feet per acre. This difference is the result of the large change in the size of the $b_{3}$ parameter between the two equations. Again, it is expected that dominant trees would exhibit low values of CCFL regardless of the value of BA. As a result, the range of their "reasonable" HCB prediction space would occupy the left-hand side of the graph.

Figure 5.13 displays predicted BR for both editions of SMC-ORGANON HCB equations plotted over CCFL for three BA values (e.g., 50, 150, and 250 square feet/acre) and with H fixed to 135 -feet, D/H fixed to 0.05 , SI fixed to 120 -feet, and OG fixed to 0.1 . The values of $H$ and $D / H$ were chosen to represent a large tree in the understory of a stand much older than 61 years based on a $\mathrm{SI}_{\mathrm{B}}$ value of 120 -feet.

Figure 5.13. Predicted Douglas-fir BR for the original and revised SMC-ORGANON HCB equations plotted over CCFL for three BA values (e.g., 50, 150, and 250 square feet/acre) and with H fixed to 135 -feet, $\mathrm{D} / \mathrm{H}$ fixed to 0.05 , SI fixed to 120 -feet, and OG fixed to 0.1 . The values of H and $\mathrm{D} / \mathrm{H}$ were chosen to represent a large tree in the understory of a stand much older than 61 years based on a $\mathrm{SI}_{\mathrm{B}}$ value of 120 -feet.


Figure 5.13 shows that the revised edition of SMC-ORGANON predicts lower values HCB for such a tree than the original edition, with the difference increasing with decreasing BA. As with shorter suppressed trees, it would be expected that taller suppressed trees growing in a stand with a high value of BA would also exhibit higher values of CCFL, and vice a versa. As a result,
the range of their "reasonable" HCB prediction space would shift with the trees' particular values of CCFL and BA.

Figure 5.14 plots predicted BR for both editions of SMC-ORGANON HCB equations over CCFL for three BA values (e.g., 50, 150, and 250 square feet/acre) and with H fixed to 135feet, D/H fixed to 0.25 , SI fixed to 120 -feet, and OG fixed to 0.1 . The values of H and $\mathrm{D} / \mathrm{H}$ were chosen to represent a large tree in the overstory of an approximately 61-yearold stand based on its $\mathrm{SI}_{\mathrm{B}}$ value of 120 -feet.

Figure 5.14. Predicted Douglas-fir BR for the original and revised SMC-ORGANON HCB models plotted over CCFL for three BA values (e.g., 50, 150, and 250 square feet/acre) and with H fixed to 135 -feet, $\mathrm{D} / \mathrm{H}$ fixed to 0.25 , SI fixed to 120 -feet, and OG fixed to 0.1 . The values of H and $\mathrm{D} / \mathrm{H}$ were chosen to represent a large tree in the overstory of an approximately 61yearold stand based on its $\mathrm{SI}_{\mathrm{B}}$ value of 120 -feet.


Inspection of Figure 5.14 shows the revised equation consistently predicts lower values of HCB than the original equation for such a tree, with the difference increasing with an increase in BA. It is expected that dominant trees would exhibit low values of CCFL regardless of the value of BA. As a result, the range of their "reasonable" HCB prediction space would occupy the left hand side of the graph.

In summary, the addition of many more trees with measured values of HCB from young stands appears to have resulted in smaller values of predicted BR (and, therefore, HCB) than predicted with the original HCB equation. This conclusion is supported by the CR data reported in Hann et al. (2006), which found that the non-SMC data in the original analysis had an average value of 0.51 for $B R$, while the SMC data had an average value of 0.12 for $B R$.

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### 6.0 Expected Behaviors for Predicted Stand Development

The expected tree-level behaviors for the four tree-level dynamic equations in ORGANON ( $\Delta \mathrm{D}$, $\Delta H, P M$, and $\triangle H C B$ ) discussed in Chapters 2 through 5 are necessary but not sufficient for guaranteeing how the resulting equations will behave when they are combined to form a "growth and yield" or, as I prefer, a "stand development" model. The following is an outline of the system of eight functions (ftn), and the tree and stand attributes used to form their predictor variables, that ORGANON uses to project an initial tree list into the future:

1. $\Delta \mathrm{D}=\mathrm{ftn}_{1}\left(\mathrm{D}_{\mathrm{s}}, \mathrm{CR}_{\mathrm{s}}, B \mathrm{BLL}_{\mathrm{s}}, B A_{\mathrm{s}}, \mathrm{SI}\right)$
2. $\Delta \mathrm{H}=\mathrm{ftn}_{2}\left(\mathrm{H}_{\mathrm{s}}, \mathrm{CR}_{\mathrm{s}}, \mathrm{CCH}_{\mathrm{s}}, \mathrm{SI}\right)$
3. $\mathrm{PM}=\mathrm{ftn}_{3}\left(\mathrm{D}_{\mathrm{s}}, \mathrm{CR}_{\mathrm{s}}, \mathrm{BALs}, \mathrm{SI}\right)$
4. $\triangle H C B=f t n_{4}\left(D_{s}, H_{s}, C C F L_{s}, B A_{S}, S I, D_{E}, H_{E}, C C F L_{E}, B A_{E}\right)$
5. $\mathrm{BAL}_{\mathrm{s}}=\mathrm{ftn}_{5}\left(\mathrm{D}_{\mathrm{s}}, \mathrm{EXPAN}\right)$
6. $B A_{s}=f \mathrm{tn}_{6}\left(\mathrm{D}_{\mathrm{s}}, \mathrm{EXPAN}_{\mathrm{s}}\right)$
7. $\mathrm{CCFL}_{\mathrm{s}}=\mathrm{ftn}_{7}\left(\mathrm{D}_{\mathrm{s}}, \mathrm{EXPAN}_{\mathrm{s}}\right)$
8. $\mathrm{CCH}_{\mathrm{s}}=\mathrm{ftn}_{8}\left(\mathrm{H}_{\mathrm{s}}, \mathrm{D}_{\mathrm{s}}, \mathrm{CR}_{\mathrm{s}}\right.$, EXPAN $\left._{\mathrm{s}}, \mathrm{SI}\right)$
9. $B A_{E}=f t_{6}\left(D_{E}, E X P A N_{E}\right)$
10. CCFL $_{E}=\operatorname{ftn}_{7}\left(\mathrm{D}_{\mathrm{E}}\right.$, EXPAN $\left._{\mathrm{E}}\right)$
11. $\mathrm{D}_{\mathrm{E}}=\mathrm{D}_{\mathrm{S}}+\Delta \mathrm{D}$
12. $\mathrm{H}_{\mathrm{E}}=\mathrm{H}_{\mathrm{S}}+\Delta \mathrm{H}$
13. EXPAN $_{\mathrm{E}}=\mathrm{EXPAN}_{\mathrm{S}} \times\left(1.0-\mathrm{PM}_{\mathrm{S}}\right)$
14. $\mathrm{HCB}_{\mathrm{E}}=\mathrm{HCB}_{S}+\triangle \mathrm{HCB}$
15. $\mathrm{CR}_{\mathrm{E}}=\mathrm{HCB}_{\mathrm{E}} / \mathrm{H}_{\mathrm{E}}=\left(\mathrm{CR}_{\mathrm{S}} \times \mathrm{H}_{\mathrm{S}}+\Delta \mathrm{H}-\Delta \mathrm{HCB}\right) /\left(\mathrm{H}_{\mathrm{S}}+\Delta \mathrm{H}\right)$
16. $\Delta \mathrm{CR}=\mathrm{CR}_{\mathrm{E}}-\mathrm{CR}_{\mathrm{S}}$
17. Subscript $S=$ Start of the growth period
18. Subscript $\mathrm{E}=$ End of the growth period

The specific equations that define $\mathrm{ft}_{1}$ through $^{\mathrm{ftn}_{4}}$ are presented in Chapters 2 through 5, respectively. The specific equations defining $\mathrm{ftn}_{5}$ through $\mathrm{ftn}_{7}$ are found in several forest modeling books such as Pretzsch (2009), Weiskittel et al. (2011), and Burkhart and Tomé (2012), while the equation defining $\mathrm{ftn}_{8}$ is found in Weiskittel et al. (2011) and Burkhart and Tome (2012). In ORGANON, $\mathrm{ftn}_{7}$ and $\mathrm{ftn}_{8}$ need species specific and region specific equations for predicting: (1) maximum crown width (MCW) of open grown trees, (2) the largest crown width (LCW) of stand grown trees, (3) the location of LCW within the tree's crown, and (4) the crown profile above LCW.

The 18 relationships found above define the complex interactions that result in predicted development of trees using SMC-ORGANON. There are a number of expected stand-level behaviors that can be used to determine if this system of complex tree-level functions is interacting well together. Currently, all of these expected behaviors concern how untreated, even-aged, pure species stands develop over "time", whether "time" is expressed explicitly, such as the value of a stand attribute at a given stand age (e.g., top height [i.e.,H40] over age), or implicitly, such as the value of a stand attribute at a given stand size (e.g., the size-density trajectory). The reality is that there has been a much longer history in forestry practice
throughout the world of planting and managing evenaged, pure species stands than other stand structures. Even if the stand development model is designed to primarily project the development of mixed species stands or unevenaged stands, a well-developed model should be able to also project evenaged, pure species stands. This process of evaluating stand-level behaviors differs from the validation process (called benchmarking in Weiskittel et al. 2011) in which predictions are compared to measured values (collected independently from the modeling data) using statistical tools in order to evaluate the accuracy and precision of the model's predictions.

The tree-level and the stand-level attributes defined above indicates the complex manner that the four basic attributes of D, H, HCB and EXPAN interact to predict these stand-level attributes over time. In the following two analyses, the behavior of the stand attributes will be expressed in terms of net values (i.e., losses due to mortality and cutting have been removed in calculating stand attributes at a given age).

### 6.1 The Bakuzis Matrix and Data Used in Its Development

An example of using this approach to evaluate predicted stand behavior is Leary's (1997) usage of the Bakuzis matrix to evaluate whether or not two stand development models for red pine met expected stand behavior. Leary (1997) identified four approaches for evaluating predicted stand-level behavior:

- First, compare the predictions from the model to the experiences of experts.
- Second, conduct a validation of the model.
- "Third, compare model predictions with law-like relationships about stand dynamics. If a model does not violate any law-like relationships, it says nothing about its accuracy or precision, i.e. its utility. If a model does violate any of several law-like relationships, the model probably has a structural deficiency. Checking models for violation of law-like relationships between stand properties thus, helps primarily to falsify models, not to confirm them."
- Fourth, compare the predictions between two or more alternative models.

Leary (1997) defined law-like relationships of stand behavior as "...those that have been found to occur repeatedly in even-aged tree monocultures...."

The Bakuzis matrix had eight rows and eight columns with the following stand attributes, expressed in metric units, that are used to define both the rows and the columns of the matrix:

1. Stand age (A)
2. Number of trees per hectare (TPH)
3. Mean height in meters (MH)
4. Basal area in square meters per hectare (BAPA)
5. Mean diameter at breast height in centimeters at 1.3 meters above ground (MD)
6. Total stem volume in cubic meters per hectare (TSVPH)
7. Current annual increment of TSVPH (CAIPH),
8. Mean annual increment of TSVPH (MAIPH).

Each cell contained a graph of the column attribute across the row attribute for low, medium and high values of site quality.

Leary (1997) noted that of the 64 relationships in the full Bakuzis matrix, only 12 qualified as law-like relationships:

1. TPH over A (Sukachev Effect)
2. TPH over MH (Spacing Percent)
3. TPH over MD (Reineke's Rule)
4. MH over A (Height-Age by Site Classes)
5. MH over MD (Height-Diameter by Site Classes)
6. BAPH over TPH (Stocking Guide Framework)
7. TSVPH over A (Volume-Age by Site Classes)
8. TSVPH over TPH (Yield-Density Effect)
9. TSVPH over MH (Eichhorn's Rule)
10. CAIPH over A (Yield Class)
11. CAIPH over BAPH (Assman's Optimal and Critical Density)
12. CAIPH over TSVPH (Langsaeter's Hypothesis)

Leary (1997) further reduced these 12 cells by dropping the Eichhorn's Rule and the three CAIPH relationships because they were related "...to final harvest volumes and volumes harvested from thinnings... " He did this to avoid complications associated with applying thinning behavior to the output of the models being evaluated. Finally, he added two relationships that he had not identified as "law-like" relationships:

1. TPH over A (Sukachev Effect)
2. TPH over MH (Spacing Percent)
3. TPH over MD (Reineke's Rule)
4. MD over A
5. MH over A (Height- Age by Site Classes)
6. MH over MD (Height-Diameter by Site Classes)
7. BAPH over A
8. BAPH over TPH (Stocking Guide Framework)
9. TSVPH over A (Volume-Age by Site Classes)
10.TSVPH over TPH (Yield-Density Effect)

These remaining ten cells from the Bakuzis matrix were used to evaluate the predictions of red pine development from two stand development models in the Lake States, STEMS and REDPINE.

I will use the same or similar 10 cells to evaluate the stand-level behavior of predictions from the second edition of SMC-ORGANON. This resulted in the usage of the following basic variables to create the stand-level attributes of interest:

1. A
2. Number of trees per acre (TPA)
3. Top height in feet ( H 40 )
4. Basal area in square feet per acre (BAPA)
5. Quadratic mean diameter at 4.5 feet above ground in inches (QMD)
6. Total stem cubic foot volume per acre (TSVPA)

Two changes that I made were to substitute QMD for MD and H 40 for MH. I did this because QMD is used in the density-size relationship of Reineke (1933) and the top height trees are most likely to be measured in Douglas-fir plantations (i.e., it is unlikely that heights on all trees required to calculate MH are measured on Douglas-fir plantations). Another change was to use the size-density relationship of QMD plotted over N instead of the Reineke's density-size relationship because size-density relationships have received much more research interest than density-size relationships.

As a result, the following ten relationships will be used to evaluate SMC-ORGANON:

1. TPA over A (Sukachev Effect)
2. H40 over A (Height- Age by Site Classes)
3. BAPA over A
4. QMD over A
5. TSVPA over A (Volume-Age by Site Classes)
6. TPA over H40 (Spacing Percent)
7. QMD over TPA (Size-Density Rule, i.e. Inverse of Reineke's Rule)
8. H40 over QMD (Height-Diameter by Site Classes)
9. BAPA over TPA (Stocking Guide Framework)
10. TSVPA over TPA (Yield-Density Effect)

Leary (1997) described the data that Professor Egolfs V. Bakuzis used to develop his matrix in the following fashion:
"The information graphed in Fig. 1 "(i.e., the Bakuzis matrix)" was collected from fully stocked plots measured in Germany for most of a rotation of Scots pine, approximately 120-140 years. Therefore, the curves in each cell represent actual dynamic equilibrium stand trajectories, i.e. a smoothed real growth series...

I make the following observations about this species and the resulting data used to form the Bakuzis matrix:

1. Scots pine is a slow growing species with maximum sizes of H and D much smaller than Douglas-fir.
2. The longevity of Scots pine is much shorter than Douglas-fir and, therefore, BA starts declining at approximately 80 years.
3. The values in the Bakuzis matrix start at 20 years since planting and, therefore, the data probably represented plots already in the phase of self-thinning.
4. The values of TPA at age 20 vary by productively, with low site quality having approximately 3,600 TPA at age 20 , the middle site quality have approximately 2,600 TPA at age 20, and the high site quality having approximately 1,400 TPA at age 20 . These densities are extremely high for 20 -year old stands and, as a result, the stands
are more than likely to be tracking the maximum size-density line as they develop over time.

The SMC-ORGANON projections were made without using the options in SMC-ORGANON of: (1) increasing PM in order to keep the projected stand on or below its maximum size-density trajectory, and (2) calibrating the $\triangle \mathrm{D}$ and HBC equations in SMC-ORGANON to the measured $\Delta \mathrm{D}$ and HBC data available for the plots used to make the projections in this evaluation.
Therefore, the predicted stand-level values are a result of just the behavior of the four treelevel dynamic equations as they interact during the predicted development of the stand over time.

### 6.2 The Douglas-fir Data Used to compare SMC-ORGANON to the Bakuzis Matrix

Because each cell in the Bakuzis matrix presents its relationship for three levels of productivity, I chose to use three control plots selected from the SMC Type I plots as a source of data for initializing SMC-ORGANON. The SMC Type I control plots were established in operational plantations at various points in plantation development and at various values of planting density. The plots with the largest value of SI, the lowest value of SI , and that plot with a value of SI closest to the midrange SI were selected for this analysis. The measurement on each plot used to initialize SMC-ORGANON was the first measurement in which all trees on the plot were above breast height. Table 6.1 contains the Bruce. (1981) SI with a base age of 50 -years from breast height, the initial stand age from seed, and the initial TPA for the three plots used to initialize SMC-ORGANON in order to conduct this comparison.

Table 6.1. The Bruce (1981) site index, the initial stand age from seed, and the initial TPA for the three plots used in SMC-ORGANON to conduct this comparison.

| Plot Type | Bruce ('1981) Site Index | Stand Age from <br> Seed | Number of Trees per Acre |
| ---: | ---: | ---: | ---: |
| Low Site Quality | $77.6-\mathrm{Feet}$ | 20 | 420 |
| Medium Site Quality | $128.6-\mathrm{Feet}$ | 18 | 472 |
| High Site Quality | $171.2-$ Feet | 9 | 524 |

My observations concerning the data described in Table 6.1 that will be used to project the development of Douglas-fir in SMC-ORGANON are:

1. Initial A declines with increasing site index (SI)
2. Initial TPA increases with increasing SI
3. Midrange value of SI across all Type I control plots was 124.4-Feet so the SI value of 128.6 -feet for the medium site quality plot used in the analysis was close to the midrange value of the Type I plots
4. The three initial TPA values used in the analysis were similar to each other but their range was much smaller than the range in initial TPA values used in the creation of the Bakuzis matrix.
5. All of the initial measurements on the three selected Type I plots used in this analysis appear to be still approaching the maximum size-density line.

### 6.3 Comparing SMC-ORGANON Output to the Bakuzis Matrix

The following graphs compare predicted behavior of stand-level attributes from SMCORGANON to the expected behaviors found in the Bakuzis matrix. Metric units of measurement are used in the Bakuzis matrix while English units of measurement are used in the SMC-ORGANON projections. However, these differences in the units of measurement are scalar differences and, therefore, they should not affect the relative shapes of the two sets of graphs. Differences in the relative shapes may be caused by underlining differences in the initial data used to produce the graphs or to possible problems with SMC-ORGANON projections.

It should also be noted that in the last five comparisons, the SMC-ORGANON graphs use predicted values for both the $y$-axis and the x-axis. Therefore, the behavior of these five graphs can be affected by the predictive behavior of either variable used to form the graph, making it more challenging to produce graphs that meet the expected behavior found in the Bakuzis matrix.

### 6.3.1. TPH over A

Figure 6.1. The Bakuzis matrix expected values of TPH plotted across A for three site qualities ( $\mathrm{I}=$ High, $\mathrm{III}=$ Medium, $\mathrm{V}=$ Low).


Figure 6.2. Predicted values of TPA from SMC-ORGANON plotted across A for three values of SI.


Once SMC-ORGANON has projected the stands past the behavior caused by both the initial differences in planting densities and the fact that the initial values of TPA and A for the three stands were not yet on the maximum size-density line and exhibiting density dependent mortality, the shape of the trend in TPA over A from SMC-ORGANON behaves as would be expected from the values of TPH over A found in the Bakuzis matrix.

### 6.3.2. MH over A

Figure 6.3. The Bakuzis matrix expected MH plotted across A for three site qualities (I = High, III = Medium, V = Low).


Figure 6.4. Predicted H40 from SMC-ORGANON plotted across A for three values of SI .


The shape of predicted H40 from SMC-ORGANON agrees with that predicted for MH from the Bakuzis matrix.

### 6.3.3. BAPH over A

Figure 6.5. The Bakuzis matrix expected BAPH plotted across A for three site qualities (I = High, III = Medium, V = Low).


Figure 6.6. Predicted BAPA from SMC-ORGANON plotted across A for three values of SI.


Predicted values of BAPA agree in the young ages but the relatively short longevity of Scots pine results in an early peak in BAPH that does not occur in the much longer-lived Douglas-fir. The difference between Figures 6.5 and 6.6 is caused by species differences and not problems with the predictions from SMC-ORGANON.

### 6.3.4. MD over A

Figure 6.7. The Bakuzis matrix expected MD plotted across A for three site qualities (I = High, III = Medium, $\mathrm{V}=$ Low).


Figure 6.8. Predicted QMD from SMC-ORGANON plotted across A for three values of SI.


The shape of predicted QMD from SMC-ORGANON agrees with that predicted for MD from the Bakuzis matrix.

### 6.3.5. TSVPH over A

Figure 6.9. The Bakuzis matrix expected TSVPH plotted across A for three site qualities (I = High, III = Medium, V = Low).


Figure 6.10. Predicted TSVPA from SMC-ORGANON plotted across A for three values of SI .


The shape of predicted TSVPA from SMC-ORGANON agrees with that predicted for TSVPH from the Bakuzis matrix.

### 6.3.6. TPH over MH

Figure 6.11. The Bakuzis matrix expected TPH plotted across MH for three site qualities ( $\mathrm{I}=$ High, III = Medium, V = Low).


Figure 6.10. Predicted TPA from SMC-ORGANON plotted across predicted H 40 for three values of SI .


Once SMC-ORGANON has projected the stands past the behavior caused by both the initial differences in planting densities and the fact that the initial values of TPA and A for the three stands were not yet on the maximum size-density line, the shape of the trend in TPA over H40 from SMC-ORGANON behaves as would be expected from the values of TPH over MH found in the Bakuzis matrix.

### 6.3.7. MD over TPH

Figure 6.13. The Bakuzis matrix expected MD plotted across TPH (i.e., the size-density relationship) for three site qualities ( $\mathrm{I}=$ High, $\mathrm{III}=$ Medium, $\mathrm{V}=$ Low ).


A clearer description of the maximum size-density trajectory in the space of QMD over TPA is illustrated in Figure 6.14 for how the four planting densities in Douglas-fir have developed in the Amance, France spacing study.

Figure 6.14. Measured QMD in centimeters (on the y-axis) across measured values of TPH for four planting spacings (on the $x$-axis) for the Douglas-fir spacing study in Amance, France, and the resulting size-density trajectory fit to the data.


Figure 6.14 is composed of: (1) a maximum size-density line shown as the concave up shaped boundary at the top of the graph, and (2) four trajectories that define the paths that QMD would follow from the initial planting densities to the maximum size density line.

The predicted SMC-ORGANON values of QMD plotted across predicted TPA for three values of SI are presented in Figure 6.15.

Figure 6.15. Predicted QMD from SMC-ORGANON plotted across predicted TPA for three values of SI.


Figures $6.13,6.14$, and 6.15 show stands developing larger values of MD or QMD as they grow from the lower right of each graph to the upper left. The Scots pine data employed to develop the Bakuzis matrix used data from heavily stocked stands (i.e, from 1,400 to 2,600 TPA), while the data employed to develop SMC-ORGANON used data from plots with much lighter stocking (i.e., from 420 to 524 TPA). The relationship of MD to TPH from the Bakuzis matrix behaves as would be expected from stands developing along the maximum size-density line of the size-density trajectory (Smith and Hann 1984), while the relationship of QMD to TPA from SMC-ORGANON behaves as would be expected from stands developing along the full size-density trajectory.

It appears in both Figure 6.13 and Figure 6.15 that the depicted plots are not approaching a common maximum-size density line which could indicate that there is a range in predicted maximum size-density trajectories for each species. Hann et al. (2003) found statistically different maximum size-density lines for plots established in both pure Douglas-fir and pure western hemlock that were measured over relatively long time periods. Furthermore, they could not find a relationship between SI and the heights of the different maximum size-density lines for Douglas-fir. Therefore, the predicted relationships found in Figure 6.15 are consistent with previous research on the size-density trajectory of Douglas-fir.

### 6.3.8. MH over MD

Figure 6.16. The Bakuzis matrix expected MH plotted across MD for three site qualities ( $\mathrm{I}=$ High, III = Medium, V = Low).


Figure 6.17. Predicted H40 from SMC-ORGANON plotted across predicted QMD for three values of SI.


The shape of predicted H 40 for a particular value of predicted QMD from SMC-ORGANON agrees with the shape of MH for a particular value of MD from the Bakuzis matrix.

### 6.3.9. BAPH over TPH

Figure 6.18. The Bakuzis matrix expected BAPH plotted across TPH for three site qualities ( $\mathrm{I}=$ High, III = Medium, V = Low).


Figure 6.19. Predicted BAPA predicted by SMC-ORGANON plotted across predicted TPA for three values of $S$.


Low initial densities, possible differences for the maximum size-density lines between the SMC Type I plots, and the relatively short longevity of Scots pine resulting in a relatively early peak in BAPH that does not occur in the much longer_-lived Douglas-fir make comparisons between the two graphs very difficult. However, both graphs predict an increase in BAPH/BAPA as TPH/TPA decreases and, initially, a greater increase in BAPA as SI increases.

### 6.3.10. TSVPH over TPH

Figure 6.20. The Bakuzis matrix expected TSVPH plotted across TPH for three site qualities (I = High, III = Medium, V = Low).


Figure 6.21. Predicted TSVPA predicted by SMC-ORGANON plotted across predicted TPA for three values of SI .


Once SMC-ORGANON has projected the stands past the behavior caused by both the initial differences in planting densities and the fact that the initial values of TPA and A for the three stands were not yet on the maximum size-density line, the shape of the trend in TSVPA over

TPA from SMC-ORGANON behaves as would be expected from the values of TSVPH over TPH found in the Bakuzis matrix.

### 6.4 Stand Relationships for Different Planting Densities

The Bakuzis matrix compares expected stand behaviors across three levels of productivity. I have found that it is also useful to examine expected stand behaviors across alternative planting densities. I will use eight stand attributes (expressed in English units) to examine their predicted stand-level behavior by projecting SMC-ORGANON for 100-years using input data for six alternative planting densities. The first five of these attributes were also used in the Bakuzis matrix, the sixth attribute was not used in the Bakuzis matric, and the last two were also used in the Bakuzis matrix but the $x$-axis and $y$-axis variables were not transformed into the $\log -\log$ space:

- TPA over A (i.e., TPA $\left.=\sum\left[E X P A N_{i}\right)\right]$, where $\sum[]=$ The sum of the value of interest across the n sample trees). Therefore, whether or not predicted TPA behaves as expected over time will depend upon the behavior of $\mathrm{ft}_{3}$.
- BAPA over A (i.e., BAPA $=\Sigma\left[0.005454154 \times \mathrm{D}_{\mathrm{i}}{ }^{2} \times \mathrm{EXPAN}_{\mathrm{i}}\right]$ ). Therefore, whether or not predicted BAPA behaves as expected over time will depend upon the combined behaviors of $\mathrm{ftn}_{1}$ and $\mathrm{ft}_{3}$.
- QMD over A (i.e., QMD $=\left[(B A P A / T P A / 0.005454154]^{0.5}\right)$ over A. Therefore, whether or not predicted QMD behaves as expected over time will also depend upon the combined behaviors of $\mathrm{ft}_{1}$ and $\mathrm{ftn}_{3}$.
- H40 over A (i.e., $\mathrm{H} 40=\Sigma\left[\mathrm{H}_{i}\right.$ for the 40 trees per acre with the largest values of $D_{i}$ on the plot]/40]. Therefore, whether or not predicted H 40 behaves as expected over time will depend upon the combined behaviors of $\mathrm{ft}_{2}$ and $\mathrm{ftn}_{3}$.
- TSVPA over A (i.e., TSVPA $=\Sigma\left[T S C F V_{i} \times\right.$ EXPAN $\left._{i}\right]$, where TSCFV $_{i}$ is the predicted total stem cubic foot volume for each sample tree on the plot). In ORGANON, TSCFV ${ }_{i}=$ $\mathrm{ft}_{9}\left(\mathrm{D}_{\mathrm{i}}, \mathrm{H}_{\mathrm{i}}, \mathrm{CR}_{\mathrm{i}}\right)$. Therefore, whether or not predicted TSV behaves as expected over time will depend upon the combined behaviors of $\mathrm{ft}_{1}, \mathrm{ftn}_{2}, \mathrm{ftn}_{3}$, and $\mathrm{ftn}_{4}$.
- Stand density index (i.e., SDI = TPA $\times[\text { QMD/10.0 }]^{1.605}$ ) over A. Therefore, whether or not predicted SDI behaves as expected over time will depend upon the behaviors of $\mathrm{fn}_{1}$ and $\mathrm{ftn}_{3}$.
- The size-density trajectory, expressed in the transformed log-log space and using QMD as the measure of size, resulting in the expression $\ln (\mathrm{QMD})=\mathrm{ftn}_{10}\left(\mathrm{TPA}, \mathrm{QMD}_{0}, \mathrm{TPA}_{0}\right)$, where $\mathrm{QMD}_{0}$ and $\mathrm{TPA}_{0}$ are the values of QMD and TPA immediately prior to the start of self-thinning. Therefore, whether or not this version of predicted SDT behaves as expected over time will also depend upon the behaviors of $\mathrm{ft}_{1}$ and $\mathrm{ftn}_{3}$.
- The Size-density trajectory, expressed in the transformed log-log space and using the mean of TSVPA as the measure of size (MTSVPA), resulting in $\operatorname{In}(M T S V P A)=$ $\mathrm{ftn}_{11}\left(\right.$ TPA, MTSVPA $A_{0}$, TPA $_{0}$ ), where MTSVPA ${ }_{0}$ is the value of MTSVPA immediately prior to the start of self-thinning. Therefore, whether or not this version of predicted SDT behaves as expected over time will depend upon the behaviors of $\mathrm{ft}_{1}$ and $\mathrm{ftn}_{9}$.

Missing from the list above is expected behavior of HCB over time. This failing is the consequence of HCB having been measured only relatively recently on research plots and, therefore, expected behavior has not been fully established.

The above order of the eight expected behaviors is not accidental. Of ultimate interest in commercial forestry is the production of some type of volume or biomass such as TSVPA, and TSVPA is related to BAPA and H 40 by the relationship TSV $=\mathrm{ftn}_{12}(\mathrm{BA}, \mathrm{H} 40)$ (e.g., Husch et al. 2003). TPA is the first relationship in the above list because it is dependent upon only one function ( $\mathrm{ftn}_{3}$ ) and because the prediction of all of the remaining relationships depends upon TPA. If the TPA relationship over $A$ is found to be reasonable, then it is assumed that $\mathrm{ftn}_{3}$ can be considered reasonably behaved within the range of $A$ examined in the analysis. If $\mathrm{ftn}_{3}$ is judged to be unreasonable, then proceeding with the evaluation is of questionable value.

If BAPA is found to be reasonable in the next evaluation, then it is assumed that $\mathrm{ftn}_{1}$ can be also considered reasonably behaved within the range of A examined in the analysis. If $\mathrm{ftn}_{1}$ is judged to be unreasonable, then proceeding with the evaluation is of questionable value.

Likewise, if H 40 is found to be reasonable, then $\mathrm{ftn}_{2}$ can also be considered to be reasonably behaved within the range of $A$ examined in the analysis. If $\mathrm{fn}_{2}$ is judged to be unreasonable, then proceeding with the evaluation is of questionable value.

Finally, if TSVPA is found to be reasonable, then $\mathrm{ftn}_{9}$ can also be considered to be reasonably behaved within the range of $A$ examined in the analysis.

The following is the output is 100-year projections of plots for six levels of planting density on the SMC Type 3 (planting density) installation 914, located on the McDonald-Dunn Research Forest of the College of Forestry, Oregon State University. The target planting densities were $100,200,300,450,700$, and 1200 TPA, with realized planting densities of 100, 212, 331, 396, 759, and 1166 TPA.

On each of the six planting densities, the initial measurement used to start SMC-ORGANON was the one in which all trees had surpassed breast height. Three of the planting densities (100, 331, and 396 TPA) had achieved this by nine-years since seed, and the other three (212, 759 , and 1166 TPA) by thirteen-years since seed. In order to make comparisons easy, it was decided to start SMC-ORGANON using the measurements at age thirteen for all plots. The Bruce (1981) site index for the installation was 138.8-feet at a breast height age of 50-years.

As before, the SMC-ORGANON projections were made without using the options in SMCORGANON of: (1) increasing PM in order to keep the projected stand on or below its maximum size-density trajectory, and (2) calibrating the $\triangle \mathrm{D}$ and HBC equations SMCORGANON to the measured $\triangle \mathrm{D}$ and HBC data available for the plots used to make the projections in this evaluation. The following are seven relationships that will be used to evaluate whether predicted stand development of the six planting densities meet expectations:

1. TPA over A
2. BAPA over A
3. H40 over A
4. QMD over A
5. TSVPA over A
6. SDI over A
7. Log of QMD over Log of TPA
8. Log of MTSVPA over Log of TPA

As with the comparisons to the Bakuzis matrix, it should also be noted that in the last two comparisons, the SMC-ORGANON graphs use predicted values for both the $y$-axis and the xaxis. Therefore, the behavior of these two graphs can be affected by the predictive behavior of either variable used to form the graph, making it more challenging to produce graphs that meet the expected behavior.

### 6.4.1. TPA over A

The following Figure 6.22 illustrates the expected behavior of TPA planted at various densities as the plots develop over A. Graph "a" is from the Wind River, Washington spacing study in Douglas-fir, and graph "b" is from the Amance, France spacing study in Douglas-fir.

Figure 6.22. Measured number of trees per acre in Douglas-fir over age for both the Wind River, Washington spacing study, graph a, and the Amance, France spacing study, graph b.


The predicted TPA planted to six densities from SMC-ORGANON over A is found in Figure 6.23.

Figure 6.23. Predicted TPA from SMC-ORGANON across A for six planting densities.


A comparison of Figure 6.23 to Figure 6.22 shows that the predictions from SMC-ORGANON meet the expected behavior.

### 6.4.2. BAPA over A

It is expected that BAPA over A will develop in a sigmoid fashion until mortality finally becomes large enough to cause it to start declining (e.g., Figures 6.5 and 6.6).

Figure 6.24. Predicted BAPA from SMC-ORGANON across A for six planting densities.


Figure 6.24 shows that predicted BAPA over A increases as expected (i.e., sigmoidally) for each of the planting densities. Whether the predicted BAPA of lower planting densities should cross the predicted BAPA of higher densities at a given value of A is dependent upon two conditions. The first is the size of the slope for the maximum size density line. A value of -0.5 for the slope of the maximum size-density line results in a constant value of maximum BAPA for all values of density on the maximum size-density line (see Appendix 6.7 for proof). A value of the slope that is greater than -0.5 [such as the Reineke (1933) slope of -0.62305 ] will result in maximum BAPA increasing as density on the maximum size-density line decreases (see Appendix 6.7 for proof).

If the slope of the maximum size-density line exceeds -0.5 , then the second condition needed for predicted BAPA of lower density to exceed the predicted BAPA of a higher density is the relative rates in the growth of QMD for different planting densities. It is expected that the size of QMD at a given age will increase as planting density decreases (see Figures 6.25 and 6.26). If, at a given age in which all planting densities are on the maximum size-density line, the resulting value of QMD for a given planting density is larger than the QMD at of a larger planting density, then the BAPA of the lower planting density will be larger than that found on the higher planting density. The results in Section 6.4 .7 shows that these two conditions are met for the SMC-ORGANON projections used in this analysis and, therefore, the values of BAPA crossing in Figure 6.24 meet expected behavior.

At this time, the crossing of BAPA for different planting densities is purely hypothetical. A number of years ago, Dr. Dave Hyink, who was the chief forest biometrician at Weyerhaeuser Company, offered a cash reward to anyone who could present data in which BAPA values for different planting densities actually crossed. As far as I know, the reward was never collected.

### 6.4.3. QMD over A

The expected behavior of QMD over stand age is illustrated in Figure 6.25. This figure was produced from data collected in a Douglas-fir spacing trial in Amance, France. The low density consistently has larger values of QMD and the highest density the small values of QMD at any age.

Figure 6.25. Measured Douglas-fir QMD across stand age for four planting spacings in Amance, France.


The predicted behavior from SMC-ORGANON of QMD over A for six planting densities is found in Figure 6.26.

Figure 6.26. Predicted QMD from SMC-ORGANON across A for six planting densities.


A comparison of Figure 6.26 to Figure 6.27 shows that the predictions from SMC-ORGANON meet the expected behavior.

### 6.4.4. H40 over A

The expected behavior of H 40 plotted across A is shown in Figure 6.3 and verified in Figure 6.4. The predicted behavior from SMC-ORGANON of H40 planted to six densities over A is found in Figure 6.27.

Figure 6.27. Predicted top height from SMC-ORGANON across age since seed for six planting densities.


The shape of the predicted H 40 curve over A shown in Figure 6.27 agrees with the expected behavior found in Figure 6.3 and Figure 6.4. A simple linear regression of H 40 at age 113 on initial planting density shows a trend with a coefficient of determination of 0.6285 (Figure 6.28), with H 40 at age 113 decreasing with increasing initial planting density. This discovery agrees with the results of Flewelling et al. (2001) who found that increasing density negatively impacted the development of H40 in Douglas-fir.

Figure 6.28. Predicted H 40 at age 113 plotted across initial planting density in TPA.


One potential "validation" that can be conducted with the predicted values of H40 over A is to compare predicted H 40 at the SI's base age to the value of SI used to make the projections in SMC-ORGANON. However, estimating H 40 for a base age of 50 -years taken at breast height from the SMC-ORGANON output is complicated by: (1) the use of total age in the SMCORGANON output instead of breast height age used in Bruce's (1981) H40 equation, (2) the use of the data taken at age 13 -years from seed to initialize the runs for the 100 -year projections, and (3) SMC-ORGANON's usage of a five year growth period. The first complication required an estimate of the number of years needed to grow to breast height. The last two complications resulted in the SMC-ORGANON output not providing direct predictions of H 40 at a breast height age of 50 -years.

Approximate estimates of H 40 at breast height age 50 were calculated by assuming that it took six-years to reach breast height and then linearly interpolating the H40 values from the SMCORGANON output for total ages of 58 and 53 -years to a total age of 56 -years. Differences between these estimates of SI and the measured SI value of 138.3 -feet used in the projections were expressed in percentages of the measured SI. The results are found in Figure 6.29 along with a simple linear regression of the percentages across initial planting density. Examination of Figure 6.29 shows that the SMC-ORGANON predictions of H40 at a breast height of 50years consistently underestimated the measured value and the differences increased with increasing planting density. The size of the differences ranged from approximately four-percent for low planting densities to a high of approximately eight-percent for the high planting densities. The decrease in predicted SI as planting density increases parallels what was found for predicted H 40 at age 113 -years from seed. The consistent under-prediction of SI across all planting densities might indicate that the current $\Delta H$ parameter estimates could potentially be improved with a more extensive data set than used to parameterize the current edition of SMC-ORGANON. However, it should be remembered that the results reported here are for only one installation in the SMC planting density study.

Figure 6.29. Percent differences between predicted H 40 at breast height age of 50 -years and the site index for the installation (e.g., 138.8-feet) plotted across initial planting density in TPA.


### 6.4.5. TSVPA over A

The expected behavior of TSVPA plotted across A is shown in Figure 6.9 and verified in Figure 6.10. The predicted behavior from SMC-ORGANON of TSVPA planted to six densities over A is found in Figure 6.28.

Figure 6.28. Predicted TSVPA from SMC-ORGANON across A for six planting densities.


The shape of the predicted H 40 curve over A shown in Figure 6.28 agrees with the expected behavior found in Figure 6.9. TSVPA should be the product of predicted BAPA times predicted H40 and, therefore, its behavior must mimic the behaviors of predicted BAPA and predicted H40 over time. Section 6.4 .2 shows that BAPA is predicted to increase sigmoidally over A, and that predicted BAPA of lower densities will cross predicted BAPA of higher densities for larger values of A . Section 6.4 .3 shows that H 40 is predicted to increase sigmoidally over A .
Therefore, it is expected that TSVPA will increase sigmoidally over A, and that predicted TSVPA of lower densities will cross predicted TSVPA of higher densities for larger values of A. Examination of Figure 6.28 shows that predicted TSVPA across planting densities meets expected behavior.

In addition to planting density, the size of TSVPA can also affected by predicted CR values of the trees over time. Two trees with the same value of D and H but different values of CR will have larger values of TSCFV for the tree with a small value of CR than the tree with large value of it. Given the complexity of the dynamics between trees over time, it is difficult to determine how much of the differences between planting densities are caused by the impact of CR upon TSCFV.

### 6.4.6. SDI over A

The expected behavior of SDI over A in pure, evenaged stands is that it will sigmoidally approach a maximum value and then remain constant until the stand begins to break up due to insects, disease, or other natural processes (Oliver and Larson 1996).

Figure 6.29. Predicted SDI from SMC-ORGANON across A for six planting densities.


For the planting density of 1166 TPA, the value of SDI dips slightly between $A=38$ and $A=53$ (with a reduction of $4.8 \%$ between these values of $A$ ), and then begins to increase again at a slow rate. This dip occurs only for the planting density of 1166 TPA, and I have found it to also occur on the highest planting density plot of the SMC planting density installation 901.
Examination of five-year QMD increment ( $\triangle Q M D$ ) values over A indicates that this dip is caused by a plateau in the rate of reduction in $\triangle$ QMD over A for both planting densities 759 TPA and 1166 TPA followed by an increase in the rate of reduction (Figure 6.30). It appears that the plateau in the rate of reduction in $\triangle$ QMD for the planting density of 759 was not severe enough to cause a dip in SDI. Figure 6.31 shows the same graph with the four lowest planting densities removed.

Figure 6.30. Predicted five-year $\triangle \mathrm{QMD}$ increment across A for the six planting densities.


Figure 6.31. Predicted five-year $\triangle \mathrm{QMD}$ increment across A for the two highest planting densities.


My suspicion is that the plateau in $\triangle$ QMD is being caused by relatively small problems with the calculation of $\triangle C R$, which involves equations for predicting $\triangle H$ and $\triangle H C B$. The data used to model $H C B$, which is then used to calculate $\triangle H C B$, did not contain plots with planting densities
as high as 759 and 1166 TPA. Unfortunately, the ORGANON yield table does not include average values of CR or HCB that could be used to check this hypothesis.

The slow increase in SDI for the planting densities between 331 and 759 TPA is caused by the slope of the maximum size-density line being steeper than the slope value for Reineke's (1933) SDI relationship (see Section 6.4.7). Projections of stand development to age 113 is extrapolating the equations beyond the modeling data used to develop the revised edition of SMC-ORGANON employed in this analysis, particularly given the lack of a $D^{2}$ term in the PM equation to increase PM in trees with larger values of $D$.

### 6.4.7. Logarithm of Predicted QMD over Logarithm of Predicted TPA (i.e., the Size-Density Trajectory \#1)

The expected behavior of the size-density trajectory (in its most commonly used log-log space and using QMD as the measure of size) as a stand develops over time is illustrated by how the four planting densities in Douglas-fir have developed in the Amance, France planting density installation (Figure 6.32).

Figure 6.32. Measured log of QMD in centimeters (on the y-axis) across measured values of log of TPH for four planting spacings (on the x-axis) for the Douglas-fir spacing study in Amance, France, and the resulting size-density trajectory fit to the data.


Figure 6.32 is composed of: (1) the log of maximum size-density line shown as the straight line defining the upper boundary, and (2) four trajectories that define the paths that log of QMD would follow from the initial log of the planting densities to the log of maximum size density
line. In comparison, Figure 6.33 shows the projected size-density trajectory from SMCORGANON.

Figure 6.33. SMC-ORGANON predicted size-density trajectory, using QMD as the measure of size, and its maximum for six planting densities [ $\ln ($ () is the natural logarithm of the value with in the parentheses].


After projecting the plots for 100-years, three of the six planting densities shown in Figure 6.33 have reached the maximum size-density line (planting densities 396, 759, and 1166 TPA). For the planting density of 1166 TPA, the average slope of the predicted maximum size-density line for the last 20 -years shown in Figure 6.33 is -0.6927 , which is $11.2 \%$ steeper than Reineke's (1933) slope of -0.6231 . Examination of the QMD values at age 113 shows that the ending values of QMD were 18.4 -inches for the planting density of 1166 TPA, 18.6 -inches for the planting density of 759 TPA, and 19.3 -inches for the planting density of 396 TPA. The effect of having a predicted slope of the maximum size-density line that is steeper than -0.5 and also having the ending values of QMD at age 113 increase with decreasing planting is to have predicted values of BA cross each other at older ages as shown in Figure 6.24.
6.4.8. Logarithm of Predicted MTSVPA over Logarithm of Predicted TPA (i.e., the Size-Density Trajectory \#2)

The expected behavior of the size-density trajectory (in its most commonly used log-log space and using MTSVPA as the measure of size) as a stand develops over time is illustrated by how the six planting densities in red pine have developed over time (Figure 6.34 from Smith and Hann 1984).

Figure 6.34. Measured $\ln$ (MTSVPH in cubic meters) across measured values of $\ln (T P H)$ for six planting spacings for red pine from the Petawawa Forest Experiment Station, Ontario, Canada (Stiell and Berry 1973), and the resulting size-density trajectory fit to the data (Smith and Hann 1984).


The predicted behavior using SMC-ORGANON of the size-density trajectory using MTSVPA as the measure of size is shown in Figure 6.35.

Figure 6.35. SMC-ORGANON predicted size-density trajectory, using MTSVPA as the measure of size, and its maximum for six planting densities.


After projecting the plots for 100 -years, two of the six planting densities shown in Figure 6.35 have reached the maximum size-density line (planting densities 759, and 1166 TPA). For the planting density of 1166 TPA, the average slope of the predicted maximum size-density line for the last 20 -years shown in Figure 6.35 is -1.9110 , which is $27.4 \%$ steeper than the $-3 / 2$ slope that has been assumed by many researchers in the past (e.g., Yoda et al. 1963, White and Harper 1970, Harper 1977, Drew and Flewelling (1979), Smith and Hann 1984). Examination of the MTSVPA values at age 113 shows that the ending values of MTSVPA were 133.9-cubic feet per acre for the planting density of 1166 TPA and 143.3-cubic feet per acre for the planting density of 759 TPA. Given the graph of TSVPA over A (Figure 6.28) shows that there are multiple crossings in predicted TSVPA at older ages for all planting densities except 100-TPA and given that MTSVPA at age 113 increases with decreasing planting density for all densities, it is likely that, as with BAPA over A, this behavior is the result of the slope of the maximum line being steep enough to prevent the maximum size-density line from approaching a slope that would predict a constant value of MTSVPA regardless of the size of the planting density.

### 6.5 Discussion

The results from Section 6.3 (i.e., the first set of figures that compared predicted stand-level behavior from SMC-ORGANON to the expected behavior displayed in the Bakuzis matrix) showed that, in nine of ten cases (the BAPA over $N$ being a possible exception), the 100-year old stand level predictions from SMC-ORGANON exhibited or began to exhibit the stand-level behaviors found in the Bakuzis matrix, which is much more frequent than the two red pine growth and yield models examined by Leary (1997). Particularly noteworthy are the positive results for at least four of the last five comparisons in which predicted attributes from SMCORGANON are used for both the $x$ and $y$ variables.

The results from Section 6.4 (i.e., the second set of figures that compared predicted standlevel behavior from SMC-ORGANON to the expected behavior arising from planting to different densities) showed that SMC-ORGANON also met expected behavior in seven of the eight comparisons examined in this evaluation, with the SDI over A comparison showing small problems with the highest planting density. Particularly noteworthy are the very positive results for the last two comparisons in which predicted attributes from SMC-ORGANON are used for both the x and y variables.

To repeat, the SMC-ORGANON projections in both sets of evaluations were made without using the option of increasing predicted PM in order to keep the projected stand on or below its maximum size-density trajectory. Furthermore, the $\triangle \mathrm{D}$ and HCB equations in SMC-ORGANON (the later used to predict $\triangle \mathrm{HCB}$ ) were not calibrated to the measurements taken over time on the plots used to initialize the SMC-ORGANON runs. Therefore, the predicted stand-level values over time are solely the result of the interactive behavior of the four tree-level dynamic equations during the predicted development of each stand over time. These four dynamic equations were fit individually without the use of procedures such as the methods of simultaneous estimation of parameters found in econometrics (e.g., Kmenta 1986), and the stand-level estimates were not manipulated by procedures such as disaggregation (Ritchie and Hann 1997) to force expected stand behavior.

SMC-ORGANON projections of 100-years duration were made in order to produce the graphs in both Sections 6.3 and 6.4. The data used to develop the revised edition of SMC-ORGANON came from two main sources: (1) installations created by the SMC cooperators themselves before the establishment of the SMC and (2) from three types of installations created by the SMC itself. The SMC created installations included their Type I (which were installed in existing young plantations), Type II (which were installed in existing older plantations), and Type III installations (which were installed on bare ground using six alternative planting densities). The maximum stand age from seed in the modeling data set for the SMC created installations was 18-years for the Type I installations, 40-years for the Type II installations, and 10-years for the Type III installations.

The maximum stand age for the modeling data from the installations created by SMC cooperators ranged from 77-years old at breast height for the $\Delta \mathrm{D}$ dataset (which required measurements of H and HCB at both the start and end of the measurement period) to 108years old at breast height for the HCB dataset (which required measurements of H and HCB at
a measurement). For the $\Delta \mathrm{D}$ data set, a total of $331 \Delta \mathrm{D}$ observations from the SMC cooperators' installations were in stands greater than 40-years old at breast height. To strengthen the $\triangle D$ data set at older ages, it was decided to add the ORGANON $\triangle D$ data sets from southwest Oregon, northwest Oregon, and western Washington to the $\Delta \mathrm{D}$ modeling dataset. The combined southwest and northwest Oregon data sets added 10,843 $\Delta \mathrm{D}$ measurements from mostly natural, evenaged stands that were greater than 40-years old at breast height (and up to an evenaged stand that was 354 years old at breast height). All of the western Washington $\Delta \mathrm{D}$ measurements and approximately $30 \%$ of the southwest Oregon $\Delta \mathrm{D}$ measurements were in unevenaged stands.

As a result, the revised edition of SMC-ORGANON contained a Douglas-fir $\Delta \mathrm{D}$ equation which was fit to data from stands with ages that ranged up to 354 years old, a $\Delta H$ equation which was fit to data from stands with ages that ranged up to 81-years old, a PM equation which was fit to data from stands that ranged up to 87-years old, and a HCB equation which was fit to data from stands that ranged up to 108-years old. Therefore, the first set of 100-year old projections presented in Section 6.3 were made with equations that were fit to data sets that approximately covered the range of stand ages resulting from that analysis. However, it should be remembered that: (1) the amount of data over 40-years of age available to fit the $\Delta \mathrm{H}, \mathrm{PM}$ and HCB equations were limited in quantity, and (2) none of the equations in ORGANON use stand or tree age as predictor variables.

As with the first analysis, the second set of 100-year projections presented in Section 6.4 were made with equations that, again, were fit to data sets coming from mostly existing natural stands and some plantations but not from purposefully created planting density studies (and particularly high planting densities). Therefore, the results found in Section 6.4 are evaluating how well the equations developed from natural stands and some plantations can extrapolate to predicting the development of plantations that were planted to alternative densities. The fact that the results presented in Section 6.4 showed that the revised edition of SMC-ORGANON could successfully project such stand structures indicates that the model forms and associated parameter estimates are well behaved up to approximately 100-years of age.

Hann and Hanus (2001) found that predicted PM started to increase in Douglas-fir after trees reached values of $D$ above 37.6 -inches. They modeled this increase in PM by including a $D^{2}$ term in their PM equation. The data used to model PM in SMC-ORGANON was inadequate in duration to include a $D^{2}$ term in the equation. The largest value of $D$ at the end of the 100-year projections in Section 6.4 was 37.3 - inches, which occurred on the 100-TPA planting density. Therefore, the length of the projections for the planting density analysis were made as long as possible without resulting in a likely underestimation of PM for large trees.

This evaluation examined the relative behavior of predictions from SMC-ORGANON to those expected from stand development "theory" of evenaged, pure species stands. As Leary (1997) noted, this process "... helps primarily to falsify models, not to confirm them." Therefore, once it has been established that a model's behavior meets expectations under these stand conditions, the next step is to validate or benchmark whether or not the model's tree-level and stand-level predictions are accurate and precise using data sets independently collected with the same measurement protocols that were employed to measure the modeling data. The
validation/benchmarking process is fraught with difficulties such as obtaining appropriate validation data sets and choosing meaningful validation statistics (Weiskittel et al. 2011).

For example, in developing the revised SMC-ORGANON equations, an attempt was made by Hann et al. (2006) to validate the $\Delta \mathrm{D}$ and $\Delta \mathrm{H}$ equations using data collected by the Swiss Needle Cast Cooperative (SNCC). The validation data set consisted of 965 measurements of $\Delta \mathrm{D}$ and 960 measurements of $\Delta \mathrm{H}$ on 27 plots that ranged in age from 6 to 24 years old at breast height and that were judged to be unaffected by the disease. To complicate the analysis further, a different definition of HCB was used in the collection of the SNCC data than that used in the modeling of SMC-ORGANON, requiring the application of a conversion equation between the two definitions of HCB. The usage of the conversion equation adds uncertainty to the validation results.

The results of that validation effort indicated that the revised $\Delta D$ equation under predicted $\Delta D$ on the SNCC plots by an average of $13 \%$ and that the revised $\Delta \mathrm{H}$ equation over predicted $\Delta \mathrm{H}$ on the SNCC plots by an average of $5 \%$. One possible explanation for some of differences found in the validation of SMC-ORGANON's $\triangle \mathrm{D}$ equation is the discovery by Wensel and Turnblom (1998) and Yeh and Wensel (2000) that $\Delta \mathrm{D}$ of Douglas-fir can vary between growth periods due to differences in the amount of precipitation received in each growth period. Most of the SMC-ORGANON modeling data was collected before the collection of the SNCC data so this scenario is a possibility. As said before, the validation/benchmarking process is fraught with difficulties. Ultimately, it was left to the end user of the revised edition of SMC-ORGANON to judge whether or not these differences were important or not given the limited nature of the SNCC validation data set. If these differences are of a concern, then the user of SMCORGANON can consider the calibration features in the model.

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### 6.7 Appendix: Examination of the Behavior of BA as TPA Decreases Along a QMD Based Maximum Size Density Line with a Slope of Either -0.5 or Reineke's Slope

The maximum size-density line, using QMD in inches as the measurement of size, can be expressed as:
$\ln \left(Q M D_{i}\right)=b_{0}-b_{1} \times \ln \left(T P A_{i}\right)$
If $b_{1}=0.5$, then the following results:

```
ln(QMD i) = b b - 0.5\times In(TPA 
2x\operatorname{ln}(QMD i})=2\times\mp@subsup{b}{0}{}-\operatorname{ln}(TP\mp@subsup{A}{i}{}
2x\operatorname{ln}(\mp@subsup{QMD}{\textrm{i}}{)})+\operatorname{ln}(\mp@subsup{TPA}{i}{})=2\times\mp@subsup{\textrm{b}}{0}{}
QMD [
0.005454154\timesQMD 2}\times\mp@subsup{\textrm{TPA}}{i}{}=0.005454154\timesEXP(2\times\mp@subsup{b}{0}{}
```

Given that $B A_{i}=0.005454154 \times$ QMD $_{i}^{2} \times T P A_{i}$

$$
\mathrm{BA}_{\mathrm{i}}=0.005454154 \times \mathrm{EXP}\left(2 \times \mathrm{b}_{0}\right)
$$

Therefore $B A A_{i}$ is a constant value of $0.005454154 \times \operatorname{EXP}\left(2 \times b_{0}\right)$ for all values of QMD $_{i}$ and $T P A_{i}$ on the maximum size-density line.

If $b_{1}=0.62305$ (the reciprocal of Reineke's (1933) value of 1.605 ), then the following results:

```
\(\ln \left(Q M D_{i}\right)=b_{0}-0.62305 \times \ln \left(T P A_{i}\right)\)
\(1.605 \times \ln \left(Q M D_{i}\right)=1.605 \times b_{0}-\ln \left(T P A_{i}\right)\)
\(1.605 \times \ln \left(Q M D_{i}\right)+\ln \left(T P A_{i}\right)=1.605 \times b_{0}\)
\(\mathrm{QMD}_{\mathrm{i}}{ }^{1.605} \times \mathrm{TPA}_{\mathrm{i}}=\mathrm{EXP}\left(1.605 \times \mathrm{b}_{0}\right)\)
TPA \(_{i}=\left(1.0 / Q M D_{i}^{1.605}\right) \times E X P\left(1.605 \times b_{0}\right)\)
\(\mathrm{QMD}_{\mathrm{i}}^{2} \times \mathrm{TPA}_{\mathrm{i}}=\left(\mathrm{QMD}_{\mathrm{i}}^{2} / \mathrm{QMD}_{\mathrm{i}}^{1.605}\right) \times \mathrm{EXP}\left(1.605 \times \mathrm{b}_{0}\right)\)
\(0.005454154 \times\) QMD \(_{i}^{2} \times \mathrm{TPA}_{\mathrm{i}}=0.005454154 \times\left(\mathrm{QMD}_{\mathrm{i}}{ }^{2} / \mathrm{QMD}_{\mathrm{i}}{ }^{1.605}\right) \times \mathrm{EXP}\left(1.605 \times \mathrm{b}_{0}\right)\)
\(\mathrm{BA}_{\mathrm{i}}=0.005454154 \times\left(\mathrm{QMD}_{\mathrm{i}}^{2} / \mathrm{QMD}_{\mathrm{i}}^{1.605}\right) \times \mathrm{EXP}\left(1.605 \times \mathrm{b}_{0}\right)\)
\(B A_{i}=0.005454154 \times\) QMD \(_{i}^{0.395} \times E X P\left(1.605 \times b_{0}\right)\)
```

Therefore $\mathrm{BA}_{i}$ increases with an increase in $\mathrm{QMD}_{\mathrm{i}}{ }^{0.395}$ when the plot is on the maximum sizedensity line.


[^0]:    ${ }^{1}$ At the time when SWO-ORGANON was developed, southwest Oregon had the only substantial forest products company in the Pacific Northwest that was using unevenaged forest practices to manage their lands.

