Revised Volume and Taper Equations for Six Major Conifer Species in Southwest Oregon
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## Revised Volume and Taper Equations for Six Major Conifer Species in Southwest Oregon

The equations used in ORGANON for predicting the various elements of tree volume and taper were published in the 1980s: bark thickness equations (Larsen and Hann 1985), stump diameter at 1.0-feet above the ground equations (Walters et al. 1985), total stem cubic foot volume above breast height equations (Walters et al. 1985, Hann et al. 1987), merchantable cubic foot volume above breast height equations (Walters and Hann 1986a), and stem taper above breast height equations (Walters and Hann 1986b). Details about the modeling data sets and model forms used in these equations can be found in their respective publications. The southwest Oregon taper equations for Douglas-fir were subsequently tested for application in northwest Oregon using a stem analysis data set collected on the Blodgett Tract of the College of Forestry Research Properties. This analysis found that the southwest Oregon equations were superior to all other publically available taper equations in the region. As a result, the southwest Oregon volume and taper equations are the default equations for conifer species in the four versions of ORGANON.

A new ORGANON study in the 1990s resulted in the collection of additional stem analysis data for Douglas-fir and white/grand firs in either old growth stands or hardwood dominated stands in southwest Oregon. The Douglas-fir data were collected for both understory trees and dominant trees and the white/grand firs data were collected for just understory trees. A description of the data collection procedures using in this study can be found in Hann and Weiskittel (2010).

Preliminary analysis of the new Douglas-fir data indicated that the volume and taper equations were biased for trees with diameters at breast height (D) over 30-inches (Hann and Weiskittel 2010). Furthermore, there have been great improvements made in the statistical tools available for nonlinear regression analysis since the 1980s. Therefore, the objective of this analysis was to refit the equations used in ORGANON for predicting the various elements of tree volume and taper.

## Total Stem Cubic Foot Volume Above Breast Height

The expanded data set available for modeling total stem cubic foot volume above breast height is $\left(\mathrm{V}_{\mathrm{abh}}\right)$ described in Table 1. Hann et al. (1987) used the following model form to characterize $\mathrm{V}_{\mathrm{abh}}$ :

$$
\begin{equation*}
V_{a b h}=b_{1} X_{1} X_{2} D^{2} H_{a b h} \tag{1}
\end{equation*}
$$

Where,

$$
\begin{aligned}
& \mathrm{X}_{1}=\left(\mathrm{H}_{\mathrm{abh}} / D\right)^{\mathrm{b} 2} \\
& \mathrm{X}_{2}=\exp \left(\mathrm{b}_{3} C R_{\mathrm{abh}}\right) \\
& \mathrm{H}_{\mathrm{abh}}=\text { Total height above breast height, in feet } \\
& \mathrm{CR}_{\mathrm{abh}}=\text { Crown ratio above breast height } \\
& \quad=H C B_{\mathrm{abh}} / \mathrm{H}_{\mathrm{abh}}
\end{aligned}
$$

$\mathrm{HCB}_{\mathrm{abh}}=$ Height above breast height to crown base, in feet
Fitting this model form was restricted by the limited capabilities of the nonlinear regression program used at that time. The following revised total stem cubic foot volume equation model form with multiplicative correction was developed by Hann and Weiskittel (2010) to correct for the over prediction bias in Equation (1):

$$
\begin{equation*}
V_{a b h}=b_{1} X_{1} X_{3} D^{2} H_{a b h} \tag{2}
\end{equation*}
$$

Where,

$$
\begin{aligned}
& X_{1}=\left(H_{a b h} / D\right)^{\mathrm{X}} \\
& X_{2}=b_{2}\left[1.0-\exp \left(b_{3} D^{\mathrm{b} 4}\right)\right]^{\mathrm{K}} \\
& X_{3}=1.0-\mathrm{b}_{7} \exp \left(-30.0[(120.0-D) / 100.0]^{30}\right)
\end{aligned}
$$

The use of a power of 30 in $\mathrm{X}_{3}$, however, makes the equation susceptible to large biases in prediction due to any possible measurement error in D. In addition, Equation (2) does not include the impact of $\mathrm{CR}_{\mathrm{abh}}$ upon $\mathrm{V}_{\mathrm{abh}}$. Therefore, the following $\mathrm{V}_{\text {abh }}$ model form was developed to minimize the potential impact of measurement error and incorporate $\mathrm{CR}_{\text {abh }}$ :

$$
\begin{equation*}
V_{a b h}=b_{1} X_{1} X_{4} X_{5} D^{2} H_{a b h} \tag{3}
\end{equation*}
$$

Where,

$$
\begin{aligned}
& X_{4}=\exp \left[\mathrm{b}_{5}\left(\mathrm{CR}_{\mathrm{abh}}\right)^{\mathrm{b} 6}\right] \\
& \mathrm{X}_{5}=\mathrm{D}^{\mathrm{b} 7}
\end{aligned}
$$

Equation (3) was fit using weighted nonlinear regression with a weight of $\left(D^{2} H_{a b h}\right)^{-2}$. The resulting parameter estimates and their standard errors are found in Table 2.

Table 1. Descriptive statistics of the data used to model total stem cubic foot volume above breast height in southwest Oregon.

| Attribute | Mean | Minimum | Maximum | Standard Deviation |
| :---: | :---: | :---: | :---: | :---: |
| Douglas-fir ( $\mathrm{n}=851$ ) |  |  |  |  |
| $\mathrm{V}_{\text {abh }}$ | 79.60 | 0.01 | 1502.98 | 164.33 |
| D | 14.85 | 0.9 | 72.0 | 10.67 |
| $\mathrm{H}_{\text {abh }}$ | 81.14 | 4.5 | 237.2 | 43.98 |
| $\mathrm{HCB}_{\text {abh }}$ | 39.53 | 0.0 | 162.0 | 29.43 |
| Incense Cedar ( $\mathrm{n}=154$ ) |  |  |  |  |
| $\mathrm{V}_{\text {abh }}$ | 11.72 | 0.0042 | 153.12 | 20.96 |
| D | 9.64 | 0.9 | 33.4 | 6.69 |
| $\mathrm{H}_{\text {abh }}$ | 37.56 | 4.2 | 114.9 | 24.08 |
| $\mathrm{HCB}_{\text {abh }}$ | 13.66 | 0.0 | 63.7 | 14.54 |
| Ponderosa Pine ( $\mathrm{n}=141$ ) |  |  |  |  |
| $\mathrm{V}_{\text {abh }}$ | 49.26 | 0.0259 | 480.03 | 68.37 |
| D | 14.41 | 1.4 | 35.6 | 7.61 |
| $\mathrm{H}_{\text {abh }}$ | 76.30 | 10.8 | 188.3 | 35.03 |
| $\mathrm{HCB}_{\text {abh }}$ | 40.82 | 2.5 | 124.0 | 23.80 |
| Sugar Pine ( $\mathrm{n}=91$ ) |  |  |  |  |
| $\mathrm{V}_{\text {abh }}$ | 62.68 | 0.0525 | 364.53 | 69.74 |
| D | 17.66 | 2.0 | 42.2 | 8.65 |
| $\mathrm{H}_{\text {abh }}$ | 83.31 | 10.0 | 170.9 | 33.87 |
| $\mathrm{HCB}_{\text {abh }}$ | 42.27 | 0.0 | 103.7 | 23.57 |
| White and Grand Firs ( $\mathrm{n}=255$ ) |  |  |  |  |
| $\mathrm{V}_{\text {abh }}$ | 43.37 | 0.0158 | 509.86 | 68.25 |
| D | 12.35 | 1.4 | 42.9 | 7.04 |
| $\mathrm{H}_{\text {abh }}$ | 71.53 | 6.6 | 157.4 | 34.10 |
| $\mathrm{HCB}_{\text {abh }}$ | 29.88 | 0.0 | 99.7 | 24.08 |

Table 2. Parameter estimates and their weighted standard errors for the southwest Oregon total stem cubic foot volume above breast height Equation (3).

| Species | Parameter Estimates and Weighted Standard Errors (in Parentheses) |  |  |  |  |  |  | k |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{b}_{1}$ | $\mathrm{b}_{2}$ | $\mathrm{b}_{3}$ | $\mathrm{b}_{4}$ | $\mathrm{b}_{5}$ | $\mathrm{b}_{6}$ | $\mathrm{b}_{7}$ |  |
| Douglas-fir | 0.00124485628 <br> (7.897305E-05) | $\begin{aligned} & 0.346490193 \\ & (0.02988941) \end{aligned}$ | $\begin{array}{r} -0.56574969 \\ (0.106573) \end{array}$ | $\begin{aligned} & 0.632239239 \\ & (0.07548751) \end{aligned}$ | $\begin{array}{r} -0.152406551 \\ (0.02079595) \end{array}$ | $\begin{array}{r} 4.55802463 \\ (1.033951) \end{array}$ | $\begin{array}{r} -0.0511867106 \\ (0.01874057) \end{array}$ | 1.0 |
| Incense Cedar | $\begin{array}{r} 0.000907632813 \\ (5.149066 \mathrm{E}-05) \\ \hline \end{array}$ | $\begin{array}{r} 0.342846727 \\ (0.03976896) \end{array}$ | $\begin{array}{r} -0.638653879 \\ (0.3054320) \\ \hline \end{array}$ | $\begin{array}{r} 1.58572040 \\ (0.7530532) \\ \hline \end{array}$ | $\begin{array}{r} 0.0 \\ \text { (NA) } \end{array}$ | $\begin{array}{r} 0.0 \\ \text { (NA) } \end{array}$ | $\begin{array}{r} 0.0 \\ \text { (NA) } \end{array}$ | 1.0 |
| Ponderosa Pine | $\begin{aligned} & 0.00127677676 \\ & (9.898660 \mathrm{E}-05) \\ & \hline \end{aligned}$ | $\begin{array}{r} 0.162198194 \\ (0.04451275) \\ \hline \end{array}$ | $\begin{array}{r} 1.0 \\ \text { (NA) } \end{array}$ | $\begin{array}{r} 0.0 \\ \text { (NA) } \end{array}$ | $\begin{array}{r} 0.0 \\ \text { (NA) } \end{array}$ | $\begin{array}{r} 0.0 \\ \text { (NA) } \end{array}$ | $\begin{array}{r} 0.0 \\ \text { (NA) } \end{array}$ | 0.0 |
| Sugar Pine | $\begin{array}{r} 0.000855844369 \\ (6.864747 \mathrm{E}-05) \\ \hline \end{array}$ | $\begin{array}{r} 0.388366991 \\ (0.04933178) \\ \hline \end{array}$ | $\begin{array}{r} 1.0 \\ \text { (NA) } \end{array}$ | $\begin{array}{r} 0.0 \\ \text { (NA) } \end{array}$ | $\begin{array}{r} 0.0 \\ \text { (NA) } \end{array}$ | $\begin{array}{r} 0.0 \\ \text { (NA) } \end{array}$ | $\begin{array}{r} 0.0 \\ \text { (NA) } \end{array}$ | 0.0 |
| White/Grand Firs | $\begin{aligned} & 0.00133397259 \\ & (0.0001086505) \\ & \hline \end{aligned}$ | $\begin{array}{r} 0.357808283 \\ (0.04253698) \\ \hline \end{array}$ | $\begin{array}{r} -0.755355039 \\ (0.08450604) \\ \hline \end{array}$ | $\begin{array}{r} 0.5 \\ \text { (NA) } \end{array}$ | $\begin{array}{r} -0.261766125 \\ (0.03518579) \\ \hline \end{array}$ | $\begin{array}{r} 1.0 \\ \text { (NA) } \\ \hline \end{array}$ | $\begin{array}{r} 0.0 \\ \text { (NA) } \\ \hline \end{array}$ | 1.0 |

## Merchantable Cubic Foot Volume Above Breast Height

The expanded data set available for modeling merchantable cubic foot volume above breast height is $\left(\mathrm{MV}_{\text {abh }}\right)$ described in Table 3. Walters and Hann (1986a) used the following model form to characterize $\mathrm{MV}_{\mathrm{abh}}$ :
$M V_{a b h}=V_{a b h}\left[1.0-b_{1}\left(T D_{\mathrm{ib}} / P D_{\mathrm{ib}}\right)^{\mathrm{b} 2}\right]$
Where,

$$
\mathrm{MV}_{\mathrm{abh}}=\text { Merchantable cubic foot volume above breast height }
$$

$\mathrm{TD}_{\mathrm{ib}}=$ Top diameter inside bark, in inches
$P D_{i b}=$ Predicted diameter at breast height inside bark, in inches
Theoretically, Equation (4) should go to zero if $\mathrm{TD}_{\mathrm{ib}}=\mathrm{PD}_{\mathrm{ib}}$ and this will only occur if $b_{1}=1.0$. Unfortunately, Walters and Hann (1986a) found $b_{1}$ to be less than 1.0 in most cases. Equation (4) also does not include an impact of crown ratio (CR) upon predicted $\mathrm{V}_{\mathrm{abh}}$. The following model form was created to address these limitations:

$$
\begin{equation*}
M V_{a b h}=V_{a b h}\left[1.0-e^{b_{1}\left(1.0-T D_{i b} / P D_{i b}\right)^{b_{2}}}\left(T D_{i b} / P D_{i b}\right)^{\left(b_{3}+b_{4} C R^{b 5}\right)}\right] \tag{5}
\end{equation*}
$$

Where,

$$
\mathrm{CR}=1.0-\mathrm{HCB} / \mathrm{H}
$$

HCB $=$ Height to crown base, in feet

$$
\mathrm{H}=\text { Total height, in feet }
$$

Equation (5) was fit using weighted nonlinear regression with a weight of $\left(\mathrm{V}_{\mathrm{abh}}\right)^{-2}$. The resulting parameter estimates and their standard errors are found in Table 4.

Table 3. Descriptive statistics of the data used to model merchantable cubic foot volume above breast height in southwest Oregon.

| Attribute | Mean | Minimum | Maximum | Standard Deviation |
| :---: | :---: | :---: | :---: | :---: |
| Douglas-fir ( $\mathrm{n}=3039$ ) |  |  |  |  |
| $\mathrm{MV}_{\text {abh }}$ | 71.49 | 0.0004 | 1502.93 | 140.94 |
| $\mathrm{D}_{\mathrm{ib}}$ | 13.46 | 0.8 | 61.9 | 8.08 |
| TD ${ }_{\text {ib }}$ | 4.76 | 0.1 | 12.0 | 3.33 |
| $\mathrm{H}_{\mathrm{abh}}$ | 90.22 | 9.3 | 252.3 | 37.96 |
| $\mathrm{HCB}_{\text {abh }}$ | 47.45 | 1.2 | 168.5 | 26.68 |
| Incense Cedar ( $\mathrm{n}=387$ ) |  |  |  |  |
| MV ${ }_{\text {abh }}$ | 16.13 | 0.0033 | 153.12 | 23.19 |
| $\mathrm{D}_{\mathrm{ib}}$ | 10.14 | 0.7 | 27.8 | 5.74 |
| $\mathrm{TD}_{\text {ib }}$ | 3.55 | 0.1 | 3.6 | 3.14 |
| $\mathrm{H}_{\text {abh }}$ | 48.41 | 4.2 | 114.9 | 24.34 |
| $\mathrm{HCB}_{\text {abh }}$ | 18.85 | 0.0 | 61.5 | 15.48 |
| Ponderosa Pine ( $\mathrm{n}=527$ ) |  |  |  |  |
| $\mathrm{MV}_{\text {abh }}$ | 47.48 | 0.0251 | 480.08 | 63.62 |
| $\mathrm{D}_{\mathrm{ib}}$ | 12.90 | 1.1 | 32.7 | 5.99 |
| TD ${ }_{\text {ib }}$ | 5.18 | 0.3 | 11.9 | 3.21 |
| $\mathrm{H}_{\mathrm{abh}}$ | 82.38 | 10.8 | 188.3 | 31.10 |
| $\mathrm{HCB}_{\text {abh }}$ | 44.77 | 2.3 | 110.5 | 21.55 |
| Sugar Pine ( $\mathrm{n}=349$ ) |  |  |  |  |
| $\mathrm{MV}_{\text {abh }}$ | 59.35 | 0.0457 | 364.50 | 64.91 |
| $\mathrm{D}_{\mathrm{ib}}$ | 15.56 | 1.7 | 37.1 | 6.61 |
| TD ${ }_{\text {ib }}$ | 5.31 | 0.3 | 11.9 | 3.33 |
| $\mathrm{H}_{\text {abh }}$ | 88.14 | 10.0 | 170.9 | 28.58 |
| $\mathrm{HCB}_{\text {abh }}$ | 45.45 | 0.0 | 101.9 | 21.33 |
| White and Grand Firs ( $\mathrm{n}=885$ ) |  |  |  |  |
| $\mathrm{MV}_{\text {abh }}$ | 45.24 | 0.0105 | 509.85 | 66.33 |
| $\mathrm{D}_{\mathrm{ib}}$ | 12.04 | 1.1 | 39.4 | 6.05 |
| TD ${ }_{\text {ib }}$ | 4.87 | 0.2 | 12.0 | 3.22 |
| $\mathrm{H}_{\text {abh }}$ | 78.52 | 6.6 | 157.4 | 31.82 |
| $\mathrm{HCB}_{\text {abh }}$ | 33.97 | 0.0 | 101.8 | 23.98 |

Table 4 Parameter estimates and their weighted standard errors for the southwest Oregon merchantable cubic foot volume Equation (5).


## Diameter Inside Bark at Breast Height

The expanded data set available for modeling diameter inside bark at breast height ( $\mathrm{D}_{\mathrm{ib}}$ ), in inches, is described in Table 5. Larsen and Hann (1985) used the following model form to characterize $\mathrm{D}_{\mathrm{ib}}$ :

$$
\begin{equation*}
D_{i b}=b_{1} D^{b 2} \tag{6}
\end{equation*}
$$

The following model form was created to incorporate the effect of $C R$ upon $D_{i b}$ :

$$
\begin{equation*}
D_{i b}=\left(b_{1} D^{b 2}\right) E X P\left[b_{3}(1.0-C R)^{0.5}\right] \tag{7}
\end{equation*}
$$

Where,

$$
\mathrm{D}_{\mathrm{ib}}=\text { Diameter at breast height inside bark, in inches }
$$

Equation (7) was fit using weighted nonlinear regression with a weight of (D) ${ }^{-2}$. The resulting parameter estimates and their standard errors are found in Table 6.

Table 5. Descriptive statistics of the data used to model diameter inside bark at breast height in southwest Oregon.

| Attribute | Mean | Minimum | Maximum | Standard Deviation |
| :---: | :---: | :---: | :---: | :---: |
| Douglas-fir ( $\mathrm{n}=1397$ ) |  |  |  |  |
| $\mathrm{D}_{\text {ib }}$ | 12.88 | 0.8 | 61.9 | 8.52 |
| D | 14.74 | 0.9 | 72.0 | 9.87 |
| H | 86.88 | 9.0 | 241.7 | 41.33 |
| HCB | 45.04 | 1.2 | 168.5 | 27.27 |
| Incense Cedar ( $\mathrm{n}=185$ ) |  |  |  |  |
| $\mathrm{D}_{\text {ib }}$ | 8.31 | 0.7 | 30.9 | 5.63 |
| D | 9.96 | 0.8 | 36.4 | 6.71 |
| H | 44.09 | 8.7 | 123.2 | 24.21 |
| HCB | 19.28 | 1.0 | 66.0 | 14.83 |
| Ponderosa Pine ( $\mathrm{n}=171$ ) |  |  |  |  |
| $\mathrm{D}_{\text {ib }}$ | 12.90 | 1.1 | 32.7 | 6.95 |
| D | 15.16 | 1.3 | 35.6 | 7.90 |
| H | 84.61 | 15.3 | 192.8 | 36.06 |
| HCB | 47.77 | 6.8 | 115.0 | 23.64 |
| Sugar Pine ( $\mathrm{n}=103$ ) |  |  |  |  |
| $\mathrm{D}_{\text {ib }}$ | 15.52 | 1.7 | 37.1 | 7.41 |
| D | 18.07 | 1.9 | 42.2 | 8.56 |
| H | 89.90 | 14.5 | 175.4 | 33.62 |
| HCB | 46.56 | 2.5 | 106.4 | 23.69 |
| White and Grand Firs ( $\mathrm{n}=342$ ) |  |  |  |  |
| $\mathrm{D}_{\text {ib }}$ | 11.08 | 1.1 | 39.4 | 6.09 |
| D | 12.29 | 1.3 | 42.9 | 6.71 |
| H | 77.47 | 11.1 | 161.9 | 33.50 |
| HCB | 37.09 | 2.1 | 106.3 | 24.03 |

Table 6. Parameter estimates and their weighted standard errors, by species, for the southwest Oregon diameter inside bark at breast height Equation (7).

| Species | Parameter Estimates and Weighted Standard Errors (in Parentheses) |  |  |
| :--- | ---: | ---: | ---: |
|  | $\mathrm{b}_{1}$ | $\mathrm{~b}_{2}$ | $\mathrm{~b}_{3}$ |
|  | 0.92443655 | 0.988866545 | -0.0341455033 |
| Douglas-fir | $(.004617388)$ | $(.001116638)$ | $(.005969868)$ |
|  | 0.878755351 | 1.0 | -0.0769605545 |
| Incense Cedar | 0.01110895 | $(\mathrm{NA})$ | 0.01965432 |
|  | 0.808600262 | 1.0174258954 | 0.0 |
| Ponderosa Pine | $(0.01023226)$ | $(0.004792178)$ | $(\mathrm{NA})$ |
|  | 0.858979036 | 1.0 | 0.0 |
| Sugar Pine | $(0.002263008)$ | $(\mathrm{NA})$ | $(\mathrm{NA})$ |
|  | 0.921624941 | 1.0 | -0.0341539565 |
| White/Grand Firs | $(0.006400245)$ | $(\mathrm{NA})$ | $(0.01023344)$ |

## Diameter Inside Bark at a 1.0-Foot Stump

The expanded data set available for modeling diameter inside bark at a 1.0 -foot stump ( ${S D_{i b}}$ ), in inches, is described in Table 7. Walters et al. (1985) used the following model form to characterize $\mathrm{SD}_{\mathrm{ib}}$ :

$$
\begin{equation*}
S D_{i b}=b_{1}+b_{2} D^{b 3} E X P\left(b_{4} C R\right) \tag{8}
\end{equation*}
$$

The following generalization of Equation (8) was used in this study:

$$
\begin{equation*}
S D_{i b}=b_{1}+b_{2} D^{b 3} E X P\left(b_{4} C R^{b 5}\right) \tag{9}
\end{equation*}
$$

Equation (9) was fit using weighted nonlinear regression with a weight of (D) ${ }^{-2}$. The resulting parameter estimates and their standard errors are found in Table 8.

Table 7. Descriptive statistics of the data used to model diameter inside bark for a one foot stump of Douglas-fir in southwest Oregon ( $n=1392$; five trees had stump heights $\neq 1.0$ ).

| Attribute | Mean | Minimum | Maximum | Standard Deviation |
| :---: | :---: | :---: | :---: | :---: |
| Douglas-fir ( $\mathrm{n}=1392$, five trees had stump heights $\neq 1.0$ ) |  |  |  |  |
| SD ${ }_{\text {ib }}$ | 14.47 | 0.9 | 68.5 | 9.71 |
| D | 14.74 | 0.9 | 72.0 | 9.84 |
| H | 86.88 | 9.0 | 241.7 | 41.33 |
| HCB | 45.04 | 1.2 | 168.5 | 27.27 |
| Incense Cedar ( $\mathrm{n}=185$ ) |  |  |  |  |
| SD ${ }_{\text {ib }}$ | 10.12 | 1.2 | 36.4 | 6.77 |
| D | 9.96 | 0.8 | 36.4 | 6.71 |
| H | 44.09 | 8.7 | 123.2 | 24.21 |
| HCB | 19.28 | 1.0 | 66.0 | 14.83 |
| Ponderosa Pine ( $\mathrm{n}=170$, one tree had a stump height $\neq 1.0$ ) |  |  |  |  |
| $S D_{\text {ib }}$ | 15.22 | 1.4 | 38.7 | 7.88 |
| D | 15.21 | 1.3 | 35.6 | 7.89 |
| H | 84.86 | 15.3 | 192.8 | 36.01 |
| HCB | 47.85 | 6.8 | 115.0 | 23.68 |
| Sugar Pine ( $\mathrm{n}=103$ ) |  |  |  |  |
| SD ${ }_{\text {ib }}$ | 18.79 | 1.9 | 44.9 | 8.87 |
| D | 18.07 | 1.9 | 42.2 | 8.56 |
| H | 89.90 | 14.5 | 175.4 | 33.62 |
| HCB | 46.56 | 2.5 | 106.4 | 23.69 |
| White and Grand Firs ( $\mathrm{n}=340$, two trees had stump heights $\neq 1.0$ ) |  |  |  |  |
| $\mathrm{SD}_{\mathrm{ib}}$ | 12.80 | 1.4 | 45.8 | 7.50 |
| D | 12.28 | 1.3 | 42.9 | 6.72 |
| H | 77.45 | 11.1 | 161.9 | 33.60 |
| HCB | 37.09 | 2.1 | 106.3 | 24.10 |

Table 8. Parameter estimates and their weighted standard errors for the southwest Oregon diameter inside bark at a one foot stump Equation (9).

| Species | Parameter Estimates and Weighted Standard Errors (in Parentheses) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{b}_{1}$ | $\mathrm{b}_{2}$ | $\mathrm{b}_{3}$ | $\mathrm{b}_{4}$ | $\mathrm{b}_{5}$ |
| Douglas-fir | $\begin{array}{r} \hline 0.149809111 \\ (0.0272635) \\ \hline \end{array}$ | $\begin{aligned} & \hline 0.900790279 \\ & (0.01238386) \\ & \hline \end{aligned}$ | $\begin{array}{r} \hline 1.0213663112 \\ (0.004316831) \\ \hline \end{array}$ | $\begin{array}{r} 0.133648456 \\ (0.01449769) \\ \hline \end{array}$ | $\begin{array}{r} 3.67532829 \\ (0.565661) \\ \hline \end{array}$ |
| Incense Cedar | $\begin{aligned} & 0.451569966 \\ & (0.06877539) \end{aligned}$ | $\begin{aligned} & 0.831752493 \\ & (0.03942509) \end{aligned}$ | $\begin{aligned} & 10560026859 \\ & (0.01665373) \end{aligned}$ | $\begin{aligned} & 0.216216295 \\ & (0.08817787) \\ & \hline \end{aligned}$ | $\begin{array}{r} 7.00446878 \\ (2.814423) \\ \hline \end{array}$ |
| Ponderosa Pine | $\begin{array}{r} 0.0 \\ \text { (NA) } \end{array}$ | $\begin{array}{r} 1.00221381 \\ (0.00396542) \\ \hline \end{array}$ | $\begin{array}{r} 0.0 \\ \text { (NA) } \\ \hline \end{array}$ | $\begin{array}{r} 0.0 \\ \text { (NA) } \end{array}$ | $\begin{array}{r} 0.0 \\ \text { (NA) } \end{array}$ |
| Sugar Pine | $\begin{array}{r} 0.0 \\ \text { (NA) } \end{array}$ | $\begin{array}{r} 1.04030514 \\ (0.006143851) \end{array}$ | $\begin{array}{r} 0.0 \\ \text { (NA) } \end{array}$ | $\begin{array}{r} 0.0 \\ \text { (NA) } \end{array}$ | $\begin{array}{r} 0.0 \\ \text { (NA) } \end{array}$ |
| White/Grand Firs | $\begin{aligned} & 0.393048214 \\ & (0.08628714) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.729932627 \\ & (0.03090854) \\ & \hline \end{aligned}$ | $\begin{array}{r} 1.0978510098 \\ (0.01293763) \\ \hline \end{array}$ | $\begin{array}{r} 0.120814754 \\ (0.01713955) \\ \hline \end{array}$ | $\begin{array}{r} 1.0 \\ \text { (NA) } \\ \hline \end{array}$ |

## Taper Above Breast Height

The expanded data set available for modeling diameter inside bark at any point on the stem above breast height ( $\mathrm{d}_{\mathrm{ib}}$ ), in inches, is described in Table 9. Walters et al. (1985) used the following taper equation model form to characterize $\mathrm{d}_{\mathrm{ib}}$ :

$$
\begin{equation*}
\mathrm{d}_{\mathrm{ib}}=\mathrm{PD}_{\mathrm{ib}}\left[\mathrm{Z}_{0}+\left(\mathrm{b}_{1,1}+\mathrm{b}_{1,2} H D+\mathrm{b}_{1,3} H \mathrm{DD}^{2}\right) \mathrm{Z}_{1}+\mathrm{b}_{2,1} \mathrm{Z}_{2}\right] \tag{10}
\end{equation*}
$$

Where,

$$
\begin{aligned}
& \mathrm{HD}=\mathrm{H}_{\mathrm{abh}} / \mathrm{D} \\
& \mathrm{Z}_{0}=1.0-\mathrm{RH}+\mathrm{I}_{2}\left(\mathrm{RH}+\mathrm{I}_{1}\left(\mathrm{JP}_{1}\left(1.0+\mathrm{JP}_{2}\right)-1.0\right)\right)-(\mathrm{RH}-1.0)\left(\mathrm{RH}-\mathrm{I}_{2} \mathrm{RH}\right) \\
& \mathrm{Z}_{1}=\left(\mathrm{I}_{2}\left(\mathrm{RH}+\mathrm{I}_{1}\left(\mathrm{JP}_{1}\left(\mathrm{RH}+(\mathrm{WLT})\left(\mathrm{JP}_{2}\right)\right)-\mathrm{RH}\right)\right)-(\mathrm{RH}-1.0)\left(\mathrm{RH}-\mathrm{I}_{2} \mathrm{RH}\right)\right) \\
& \mathrm{Z}_{2}=\mathrm{I}_{2}\left(\left(\mathrm{RH}^{2}\right)+\mathrm{I}_{1}\left(\left(\mathrm{JP}_{1}\right)(\mathrm{WLT})\left(2.0 \mathrm{RH}-\mathrm{WLT}+(\mathrm{WLT})\left(\mathrm{JP}_{2}\right)\right)-\mathrm{RH}^{2}\right)\right) \\
& \mathrm{RH}=\mathrm{h}_{\mathrm{abh}} / \mathrm{H}_{\mathrm{abh}} \\
& \mathrm{~h}_{\mathrm{abh}}=\text { Height above breast height to the d }{ }_{\mathrm{ib}} \text { value of interest, in feet } \\
& \mathrm{WLT}=(\mathrm{aHCB}-4.5) / \mathrm{H}_{\mathrm{abh}} \\
& \mathrm{I}_{1}=0.0 \text { when } 0.0 \leq \mathrm{RH} \leq \mathrm{WLT},=1.0 \text { when WLT<RH} \leq 1.0 \\
& \mathrm{I}_{2}=0.0 \text { when } \mathrm{WLT} \leq 0.0,=1.0 \text { when } 0.0<\mathrm{WLT} \\
& \mathrm{JP}=(R H-1.0) /(\mathrm{WLT}-1.0)
\end{aligned}
$$

$$
\mathrm{JP}_{2}=(\mathrm{WLT}-\mathrm{RH}) /(\mathrm{WLT}-1.0)
$$

Table 9. Descriptive statistics of the data used to model diameter inside bark stem taper above breast height of Douglas-fir in southwest Oregon ( $\mathrm{n}=1397$ ).

| Attribute | Mean | Minimum | Maximum | Standard Deviation |
| :---: | :---: | :---: | :---: | :---: |
| Section-level Attributes |  |  |  |  |
| Douglas-fir ( $\mathrm{n}=9638$ ) |  |  |  |  |
| $\mathrm{D}_{\text {ib }}$ | 9.96 | 0.1 | 61.9 | 8.18 |
| $\mathrm{habh}^{\text {a }}$ | 48.86 | 0.0 | 236.5 | 42.80 |
| Incense Cedar ( $\mathrm{n}=887$ ) |  |  |  |  |
| $\mathrm{D}_{\text {ib }}$ | 5.89 | 0.1 | 30.9 | 4.83 |
| habh | 23.93 | 0.0 | 114.8 | 22.37 |
| Ponderosa Pine ( $\mathrm{n}=1419$ ) |  |  |  |  |
| $\mathrm{D}_{\mathrm{ib}}$ | 9.26 | 0.3 | 32.7 | 6.18 |
| habh | 43.82 | 0.0 | 186.5 | 34.80 |
| Sugar Pine ( $\mathrm{n}=980$ ) |  |  |  |  |
| $\mathrm{D}_{\mathrm{ib}}$ | 10.17 | 0.3 | 37.1 | 6.53 |
| habh | 46.36 | 0.0 | 167.0 | 34.50 |
| White/Grand Firs ( $\mathrm{n}=2477$ ) |  |  |  |  |
| $\mathrm{D}_{\text {ib }}$ | 8.46 | 0.1 | 39.4 | 6.22 |
| habh | 41.66 | 0.0 | 154.5 | 34.17 |
| Tree-level Attributes |  |  |  |  |
| Douglas-fir ( $\mathrm{n}=1395$ ) |  |  |  |  |
| D | 14.74 | 0.9 | 72.0 | 9.87 |
| $\mathrm{H}_{\text {abh }}$ | 82.38 | 4.5 | 237.2 | 41.33 |
| HCB | 45.04 | 1.2 | 168.5 | 27.27 |
| Incense Cedar (n=185) |  |  |  |  |
| D | 9.96 | 0.8 | 36.4 | 6.71 |
| $\mathrm{H}_{\text {abh }}$ | 39.59 | 4.2 | 118.7 | 24.21 |
| HCB | 19.28 | 1.0 | 66.0 | 14.83 |
| Ponderosa Pine ( $\mathrm{n}=171$ ) |  |  |  |  |
| D | 15.16 | 1.3 | 35.63 | 7.90 |
| $\mathrm{H}_{\text {abh }}$ | 80.11 | 10.8 | 188.3 | 36.06 |
| HCB | 47.77 | 6.8 | 115.0 | 23.64 |
| Sugar Pine ( $\mathrm{n}=103$ ) |  |  |  |  |
| D | 18.07 | 1.9 | 42.2 | 8.56 |
| $\mathrm{H}_{\text {abh }}$ | 85.40 | 10.0 | 170.9 | 33.62 |
| HCB | 46.56 | 2.5 | 106.4 | 23.69 |
| White/Grand Firs ( $\mathrm{n}=342$ ) |  |  |  |  |
| D | 12.29 | 1.3 | 42.9 | 6.71 |
| $\mathrm{H}_{\text {abh }}$ | 72.97 | 6.6 | 157.4 | 33.50 |
| HCB | 37.09 | 2.1 | 106.3 | 24.03 |

However, Equation (10) produced biased and imprecise predictions for D greater than 30 -inches. After testing dozens of alternatives, the following model form was judged to be the best:

$$
\begin{equation*}
d_{i b}=P D_{i b}\left(Z_{0}+X_{1} Z_{1}+X_{2} Z_{2}\right) \tag{11}
\end{equation*}
$$

Where,

$$
\begin{aligned}
& X_{1}=b_{1,1}+b_{1,2} \operatorname{EXP}\left(b_{1,3} H D^{2}\right) \\
& X_{2}=b_{2,1}
\end{aligned}
$$

Equation (11) was fit using weighted nonlinear regression with a weight of $\left(P_{i b}\right)^{-2}$. The resulting parameter estimates and their standard errors are found in Table 10. Comprehensive testing of Equation (11) using all of the undamaged and damaged conifer trees measured in southwest Oregon indicated that the following restrictions should be placed on the application of the equations to minimize prediction problems that were associated with damaged trees:

$$
\begin{aligned}
& \text { If } \mathrm{HD}<3.0, \mathrm{HD}=3.0 \\
& \text { If } \mathrm{BR}>0.85, \mathrm{BR}=0.85
\end{aligned}
$$

Table 10. Parameter estimates and their weighted standard errors for the southwest Oregon above breast height diameter inside bark stem taper Equation (11).

| Species |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
|  | $\mathrm{b}_{1,1}$ | $\mathrm{~b}_{1,2}$ | $\mathrm{~b}_{1,3}$ | $\mathrm{~b}_{2,1}$ | a |
| Douglas-fir | -0.550298007 | -0.69479837 | -0.0613100423 | 0.356974513 |  |
|  | $(0.008024195)$ | $(0.03056953)$ | $(0.003031063)$ | $(0.01659491)$ | 0.50 |
| Ponderosa Pine | -0.596278066 | -0.839878829 | -0.0685768402 | 0.134178717 |  |
|  | $(0.04589183)$ | $(0.06448906)$ | $(0.01207676)$ | $(0.05093567)$ | 0.71 |
| Sugar Pine | -0.595823501 | -1.25803662 | -0.13867406 | 0.0998711245 |  |
|  | $(0.02169944)$ | $(0.354493)$ | $(0.02000517)$ | $(0.04726353)$ | 0.60 |

A comparison of the mean differences (predicted $\mathrm{d}_{\mathrm{ib}}$ minus actual $\mathrm{d}_{\mathrm{ib}}$ ) and the associated mean square errors from the Douglas-fir taper equations of Walters and Hann (1986), Hann and Weiskittel (2010), and Equation (11) are presented in Table 11. Equation (11) exhibits lower bias and higher precision than the original equation of Walters and Hann (1986), which meets the objective of this analysis. When compared to the equation of Hann and Weiskittel (2010),

Equation (11) exhibits lower bias and higher precision for trees with $D$ values under 30-inches, very similar statistics for trees with D values between 30 and 40-inches, and higher bias and lower precision for trees with D values over 40inches. The following two facts also influenced the decision of which equation to use in ORGANON:

1. Most management scenarios will produce trees with D values under 40inches.
2. Only 34 of the 1395 sample trees had D values over 40.0-inches.

Based upon this information it was decided to use Equation (11) instead of the equation of Hann and Weiskittel (2010).

Table 11. Comparison of mean differences and mean square errors from the Douglas-fir taper equations of Walters and Hann (1986), Weiskittel and Hann (2010), and Equation (11).

| DBH Classes in Inches | Number of Trees | Walters and Hann (1986) | Hann and Weiskittel (2010) | Equation (11) |
| :---: | :---: | :---: | :---: | :---: |
| (Predicted $\mathrm{d}_{\mathrm{ib}}$ ) - (Actual $\mathrm{d}_{\mathrm{ib}}$ ) in Inches |  |  |  |  |
| 0.1 to 10.0 | 534 | 0.0 | 0.0 | 0.0 |
| 10.1 to 20.0 | 527 | 0.0 | 0.0 | 0.0 |
| 20.1 to 30.0 | 232 | -0.1 | 0.0 | 0.0 |
| 30.1 to 40.0 | 72 | 0.3 | 0.1 | 0.1 |
| 40.1 to 50.0 | 24 | 0.5 | 0.0 | 0.5 |
| $50.1+$ | 10 | 0.7 | -0.3 | 0.3 |
| All | 1395 | 0.1 | 0.0 | 0.0 |
| Mean Square Error in Inches ${ }^{2}$ |  |  |  |  |
| 0.1 to 10.0 | 534 | 0.07 | 0.12 | 0.06 |
| 10.1 to 20.0 | 527 | 0.26 | 0.31 | 0.21 |
| 20.1 to 30.0 | 232 | 0.67 | 0.79 | 0.61 |
| 30.1 to 40.0 | 72 | 2.23 | 1.69 | 1.71 |
| 40.1 to 50.0 | 24 | 4.87 | 3.28 | 3.57 |
| 50.1 + | 10 | 8.27 | 3.91 | 6.48 |
| All | 1395 | 0.88 | 0.71 | 0.70 |

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