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Predicting Merchantable Volume in Cubic Feet to a Variable Top and in Scribner Board Feet to a 6-inch Top for Six Major Conifers of Southwest Oregon

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Contents

Abbreviations

BA	basal area outside bark at breast height in square feet
CB	height to live crown base in feet
CR	crown ratio
CR_{abh}	crown ratio above breast height
DIB	diameter inside bark at breast height (4.5 feet) in inches
$\widehat{DIB}$	estimated diameter inside bark at breast height in inches
dib_m	merchantable top diameter inside bark in inches
$dib_{1.0}$	diameter inside bark at 1.0 foot in inches
$\widehat{dib}_{1.0}$	estimated diameter inside bark at 1.0 foot in inches
DOB	diameter outside bark at breast height in inches
H	total tree height in feet
H_{abh}	total height above breast height in feet
h_s	stump height in feet
V_{abh}	total stem volume above breast height in cubic feet
$\hat{V}_{abh}$	estimated total stem volume above breast height in cubic feet
V_{bbh}	volume below breast height in cubic feet
V_m	merchantable volume in cubic feet to the dib_m top diameter
V_{m-abh}	merchantable volume above breast height in cubic feet
V_6	estimated merchantable volume from a 6-inch stump to a 6-inch top diameter in cubic feet
V_S	volume in Scribner board feet
V_{S-616}	volume in Scribner board feet for a 16-foot log with a 6-inch diameter inside bark
V_{S-632}	volume in Scribner board feet for a 32-foot log with a 6-inch diameter inside bark
$\hat{V}_t$	estimated total stem volume in cubic feet
ξ	random error

British/Metric Conversion

1 inch (in.) = 2.54 centimeters (cm)
1 foot (ft) = 0.3048 meter (m)
1 cubic foot (ft^3) = 0.0283 cubic meter (m^3)

Abstract

A volume ratio approach is used to predict volume in cubic feet from breast height to any top diameter inside bark from zero to 6 inches. In addition to total height and diameter outside bark at breast height, crown ratio can be an independent variable for predicting merchantable volume in cubic feet for Douglas-fir and grand or white fir. An equation, which assumes that the true shape of the lower bole is a neiloid frustrum, is presented to estimate volume from breast height to any lower stump height. Equations are included for predicting volume in Scribner board feet to a 6-inch top diameter inside bark for 16- and 32-foot log lengths. Appended tables show volume in cubic feet and in Scribner board feet for various top diameters inside bark of six major conifers.

Introduction

Equations predicting total stem volume in cubic feet are useful to foresters, but often other values are needed. Commonly, volume in cubic feet or in Scribner board feet from a lower stump height (h_s) to a merchantable top diameter inside bark (dib_m) also is significant. The objective of this study was to develop equations estimating (1) merchantable volume in cubic feet (V_m) from a variable h_s to a variable dib_m and (2) volume in Scribner board feet (V_S) from a 0.5-foot h_s to a 6-inch dib_m.

To meet the requirements for variable h_s and dib_m in the first equation, separate equations were produced to predict volume in the portions of the stem above and below breast height (4.5 feet).

There are different approaches to calculating merchantable volume for the stem above breast height (V_{m-abh}). One way is to develop equations for each potentially merchantable top diameter. This method can be time-consuming, incomplete, and, if the equations are improperly controlled, inconsistent. For example, the volume predicted to a 6-inch dib_m may exceed that predicted to a 5-inch dib_m. Also, because merchantability standards sometimes change rapidly, equations obtained with this approach may soon become obsolete (Cao et al. 1980).

A better method is to develop an equation that incorporates a variable dib_m. One way is to predict the ratio of V_{m-abh} to total stem volume above breast height (V_{abh}) and to multiply this ratio by a standard equation for V_{abh}. This is a relatively simple method of estimating merchantable volume to any dib_m. In this study, a volume ratio equation developed by Burkhart (1977) was used to predict V_{m-abh}.

To estimate volume below breast height (V_{bbh}), an equation is provided to predict volume between breast height and h_s. It is assumed that this section of the stem is shaped like a neiloid frustrum. The equations for volume above and below breast height can then be added together to obtain an estimate of total merchantable volume from h_s to dib_m (V_m).

Because many foresters in southwest Oregon measure volume in Scribner board feet, equations are provided for converting V_m to volume in Scribner board feet (V_S) to a 6-inch dib_m for both 16-foot (V_{S-616}) and 32-foot (V_{S-632}) log lengths. These equations were first developed by Chambers and Foltz (1979) in a supplement to the second edition of Comprehensive Tree-Volume Tarif Tables (Turnbull et al. 1972).

The species examined include Douglas-fir (Pseudotsuga menziesii [Mirb.] Franco), grand fir (Abies grandis [Dougl. ex D. Don] Lindl.), white fir (Abies concolor [Gord. & Glend.] Lindl.), ponderosa pine (Pinus ponderosa Dougl. ex Laws.), sugar pine (Pinus lambertiana Dougl.), and incense-cedar (Calocedrus decurrens Torr.). These are the major commercial tree species in the second-growth stands of southwest Oregon.

Data Collection

The data were collected as part of the FIR (Forestry Intensified Research) Growth and Yield Project in mixed-conifer second-growth stands of southwest Oregon. Trees that showed no severe stem or top damage during the last 5 years were selected for sampling. The selected 1,236 trees were felled to a stump height of 1.0 foot and sectioned. Total height (H) of each tree was measured after felling; height to live crown base (CB) was measured before felling. This measurement was later adjusted to the more accurate felled tree height so the crown measurement would more closely approximate that obtained under standard inventory conditions. Trees were sectioned at 4.5 feet (breast height) and thereafter at approximate 8.4-foot intervals. At each section, diameters inside and outside the bark (DIB and DOB) were measured for the longest and shortest axes, and the average diameter of the two axes was calculated with the geometric mean. This method yields the correct cross-sectional area for both ellipses and circles (Brickell 1976). The volume below breast height of each tree was calculated as the volume of a neiloid frustrum (Husch et al. 1982). The sample included a wide range of diameters and heights for each species and represented dominant, codominant, and intermediate crown classes (Table 1).

The volume of each 8.4-foot section between breast height and the beginning of the last 5-year increment in height growth was calculated with either Smalian's or Newton's formula or with a generalized prismoidal formula (Wensel 1977). Both Smalian's and Newton's formulae accurately measure this volume, although Newton's formula is considered more exact (Husch et al. 1982). Newton's formula requires three equidistant diameter measurements and, therefore, could only be used by combining two adjacent 8.4-foot sections. Volume was calculated with the generalized prismoidal formula when adjacent sections were of unequal lengths. Smalian's formula was used for any remaining odd-numbered sections and for adjacent sections whose lengths were so different that the generalized prismoidal formula was unsuitable.

TABLE 1.

DATA SUMMARY BY SPECIES.

Species	Number of trees	DOB (in.) Mean	DOB (in.) Range	Total height (ft) Mean	Total height (ft) Range
Douglas-fir	680	13.6	0.9–43.2	81.5	9.0–200.9
Grand fir	92	13.0	1.3–42.9	79.2	11.1–161.7
White fir	92	13.9	3.1–37.7	81.8	21.7–161.9
Ponderosa pine	139	14.5	1.3–35.6	81.4	15.3–192.8
Sugar pine	92	17.6	1.9–42.2	87.4	14.5–175.4
Incense-cedar	141	10.1	0.8–33.4	44.4	8.7–119.4

Volumes to 2-, 4-, and 6-inch dib_m were obtained by interpolating between a section's two end diameters, assuming the shape is parabolic, and calculating the merchantable portion with Smalian's formula. The volume of the tip section, the section between the tree tip and the beginning of the last 5-year increment in height growth, was calculated with the formula for volume of a cone. Volumes of individual sections were then summed to obtain the calculated total stem and merchantable volumes for each tree.

Data Analysis

Estimating Merchantable Volume in Cubic Feet

Volume Above Breast Height

The volume ratio equation examined in this study was first developed by Burkhart (1977):

$$V_{m-abh}/V_{abh} = 1.0 - a_1 (dib_m^{a_2}/DIB^{a_3}) + \xi \quad [1]$$

where:

V_{m-abh} = merchantable volume in cubic feet from breast height to a specified top diameter (dib_m)

V_{abh} = total stem volume above breast height in cubic feet

dib_m = merchantable top diameter inside bark

DIB = diameter inside bark at breast height

ξ = random error.

Estimates of V_{abh} can be obtained from equations previously developed by Walters et al. (1985). These equations and their coefficients are presented in Appendix A. An equation estimating DIB from DOB (Larsen and Hann 1985) and the appropriate coefficients are given in Appendix B.

Theoretically, V_{m-abh}/V_{abh} should equal zero when dib_m = DIB. One way to constrain Equation [1] to guarantee this result is to set $a_2 = a_3$ and $a_1 = 1.0$. To test whether $a_2 = a_3$, the following reduced equation was fit:

$$V_{m-abh}/V_{abh} = 1.0 - a_1 (dib_m/DIB)^{a_2} + \xi \quad [2]$$

An $\underline{F}$-test was used to test for a significant increase in lack of fit (Neter et al. 1983). Then a_1 was evaluated for a significant difference from 1.0 with a $\underline{t}$-test.

An estimate of V_{m-abh} is obtained by multiplying the predicted ratio from either Equation [1] or [2] by an estimate of V_{abh}.

Volume Below Breast Height

Because stem form in the lower bole is reportedly neiloidic (Husch et al. 1982), the equation for the volume of a neiloid frustrum is ideal for estimating the volume of the lower stem. Walters and Hann (in press) present a taper equation for estimating lower stem diameters. Integrating their equation provided the following V_{bbh} equation, which incorporates a variable stump height:

$$V_{bbh} = K_1 \, [K_2 - K_3 \, (K_4 - K_5 + K_6 - K_7)] \quad [3]$$

where:

V_{bbh} = volume between a variable stump height and breast height

$K_1 = 0.25 \, \pi \, dib_{1.0}^{\,2}$

$K_2 = [1.0/43{,}904] \, [729 + 81 \cdot R + 297 \cdot R^2 + 265 \cdot R^3]$

$K_3 = 1.0/6{,}174$

$K_4 = [4.5 - R]^3 \, h_s$

$K_5 = 1.5 \, [4.5 - R]^2 [1.0 - R] \, h_s^{\,2}$

$K_6 = [4.5 - R] \, [1.0 - R]^2 \, h_s^{\,3}$

$K_7 = [1.0 - R]^3 \, h_s^{\,4}$

$R = (DIB/dib_{1.0})^{2/3}$

$dib_{1.0}$ = diameter inside bark at 1.0 foot

DIB = diameter inside bark at breast height

h_s = variable stump height.

As previously mentioned, Appendix B contains an equation for estimating DIB. Equations estimating $dib_{1.0}$ (Walters et al. 1985) are included in Appendix C (Equations [C.1] and [C.2]).

Total Merchantable Volume

V_m can be obtained by:

$$V_m = (V_{m-abh}/V_{abh}) \cdot V_{abh} + V_{bbh} \quad [4]$$

where:

V_{m-abh}/V_{abh} = ratio of merchantable total stem volume (from Equation [2])

V_{abh} = total stem volume in cubic feet (from Equation [A.1] or [A.2], Appendix A)

V_{bbh} = volume below breast height in cubic feet (from Equation [3]).

Upper stem volume between any two top diameters can be obtained by the appropriate subtraction.

Estimating Volume in Scribner Board Feet

Chambers and Foltz (1979) present equations converting estimated merchantable volume in cubic feet for a 6-inch dib_m from a 6-inch stump (V_6) to volume in Scribner board feet from a 6-inch stump to a 6-inch dib_m for a 16-foot log (V_{S-616}):

$$V_{S-616} = \hat{V}_6 \cdot BCU1 \quad [5]$$

where:

$\hat{V}_6$ = estimated merchantable volume in cubic feet to a 6-inch top diameter from a 6-inch stump

$BCU1 = 10^x$

$x = 0.174439 + 0.117594 \, [LOG_{10}(DOB)] \cdot [LOG_{10}(B4)] - 8.210585/DOB^2 + 0.236693 \cdot [LOG_{10}(B4)] - 0.00001345 \cdot B4^2 - 0.00001937 \cdot DOB^2$

DOB = diameter outside bark at breast height

B4 = TARIF/0.912733

$TARIF = [0.912733 \cdot \hat{V}_t]/[(1.0330 \cdot (1.0 + 1.382937 \cdot EXP(-4.015292 \cdot (DOB/10)))) \cdot (BA + 0.087266) - 0.174533]$

$\hat{V}_t$ = estimated total stem volume in cubic feet

BA = basal area outside bark at breast height $= 0.005454154 \cdot DOB^2$.

$\hat{V}_6$ and $\hat{V}_t$ can be obtained by:

$$\hat{V}_6 = V_{6-abh} + V_{bbh} \quad [6]$$

$$\hat{V}_t = V_{abh} + V_{bbh} \quad [7]$$

where:

$V_{6\text{-}abh}$ = volume above breast height to a 6-inch dib_m (obtained by multiplying the prediction from Equation [2] by V_{abh})

V_{bbh} = volume below breast height above a 6-inch stump (obtained from Equation [3])

V_{abh} = total stem volume above breast height (obtained from Equation [A.1] or [A.2], Appendix A).

The equation converting V_{S-616} to a 32-foot log length (V_{S-632}) is:

$$V_{S-632} = V_{S-616} \cdot BF3216 \qquad [8]$$

where:

$$BF3216 = 1.001491 - 6.924097/TARIF + 0.00001351 \cdot DOB^2.$$

Results

The assumption that $a_2 = a_3$ was supported by the F-test comparing Equations [1] and [2]. Therefore, Equation [1] can be reduced to Equation [2] without any statistically significant loss of accuracy. The assumption that $a_1 = 1.0$ was true only for ponderosa pine and sugar pine. For these two species, a_1 was set equal to 1.0. The regression coefficients and mean square error for Equation [2] are given in Table 2.

TABLE 2.

REGRESSION COEFFICIENTS AND MEAN SQUARE ERROR FOR ESTIMATING V_{m-abh}/V_{abh}, EQUATION [2].

Species	Regression coefficients a_1	a_2	Mean square error
Using V_{abh} with crown ratio, Equation [A.2]			
Douglas-fir	0.923501	3.78681	0.0102241
Grand/white fir	0.840783	3.51758	0.0109361
Using V_{abh} without crown ratio, Equation [A.1]			
Douglas-fir	0.930057	3.74152	0.0112020
Grand/white fir	0.857067	3.40372	0.0145529
Ponderosa pine	1.00	3.46148	0.0153136
Sugar pine	1.00	3.80410	0.0092571
Incense-cedar	0.885038	3.29655	0.0161142

Tables of estimated V_m from a stump height of 0.5 foot to merchantable 4-, 5-, and 6-inch top diameters are given in Appendices D through I. In Appendices D through F, V_{m-abh} was calculated with Equation [2] by using Equation [A.1] to estimate V_{abh} and Equation [C.2] to estimate $dib_{1.0}$. These tables, therefore, show volume in cubic feet for Douglas-fir and grand or white fir across DOB, H, and CR classes. In Appendices G through I, V_{m-abh} was calculated with Equation [2] by using Equation [A.2] to estimate V_{abh} and Equation [C.1] to estimate $dib_{1.0}$. These tables present V_m for all species across only DOB and H classes.

Appendix J shows estimated V_{S-632} for Douglas-fir and grand or white fir. The estimates of total stem volume in cubic feet, which are embedded in these V_S estimates, are based on V_{abh} and $dib_{1.0}$ equations that include CR (Equations [A.2] and [C.2], respectively). Appendix J, therefore, contains tables for each species by CR classes. Appendix K also contains tables of these V_S estimates for all species, but the estimates of total stem volume in cubic feet, which are embedded in these estimated volumes, are based on V_{abh} and $dib_{1.0}$ equations without CR (Equations [A.1] and [C.1], respectively).

Summary

There are two basic conditions that theoretically should be met by an equation predicting V_m:

(1) When dib_m equals DIB, V_{m-abh} should equal 0.0.

(2) When dib_m equals 0.0, V_{m-abh} should equal V_{abh}.

The first condition is not very important, because merchantable diameters that are very close to DIB will not concern many foresters. Equation [1] was originally developed to predict the total merchantable volume of the stem and is not constrained to satisfy the first condition. The reduced Equation [2] is constrained to satisfy this condition only when $a_1 = 1.0$ (only ponderosa pine and sugar pine satisfy the first condition). Equation [2] may yield distorted predictions of V_m when dib_m approaches DIB, even when $a_1 = 1.0$, because the simplicity of the equation form may not allow extrapolation to data extremes not found in the modeling data set. For most purposes, this shortcoming will be inconsequential. The second condition, which is more important, is satisfied by Equation [2].

Equation [2] is quite simple and is compatible with previously published equations for volume (Walters et al. 1985). It is, therefore, capable of including crown measurements for Douglas-fir and grand or white fir.

Previously published equations are used to convert V_m to V_S for 16- and 32-foot log lengths with 6-inch dib_m (Chambers and Foltz 1979).

Literature Cited

BRICKELL, J.E. 1976. Bias and precision of the Barr and Stroud dendrometer under field conditions. USDA Forest Service Intermountain Forest and Range Experiment Station, Ogden, Utah. Research Paper INT-186. 46 p.

BURKHART, H.E. 1977. Cubic-foot volume of loblolly pine to any merchantable top limit. Southern Journal of Applied Forestry 1(2):7-9.

CAO, Q.V., H.E. BURKHART, and T.A. MAX. 1980. Evaluation of two methods for cubic-volume prediction of loblolly pine to any merchantable limit. Forest Science 26:71-80.

CHAMBERS, C.J., Jr., and B.W. FOLTZ. 1979. The tarif system—revisions and additions. State of Washington, Department of Natural Resources, Olympia. Note No. 27. 8 p.

HUSCH, B., C. MILLER, and T.W. BEERS. 1982. Forest mensuration. Third edition. John Wiley and Sons, New York. 402 p.

LARSEN, D.R., and D.W. HANN. 1985. Equations for predicting diameter and squared diameter inside bark at breast height for six major conifers of southwest Oregon. Forest Research Laboratory, Oregon State University, Corvallis. Research Note 77. 4 p.

NETER, J., W. WASSERMAN, and M.H. KUTNER. 1983. Applied linear regression models. First edition. Irwin, Homewood, Illinois. 547 p.

TURNBULL, K.J., G.R. LITTLE, and G.E. HOYER. 1972. Comprehensive tree-volume tarif tables. Second edition. State of Washington, Department of Natural Resources, Olympia.

WALTERS, D.K., and D.W. HANN. In press. Taper equations for six conifer species in southwest Oregon. Forest Research Laboratory, Oregon State University, Corvallis. Research Bulletin.

WALTERS, D.K., D.W. HANN, and M.A. CLYDE. 1985. Equations and tables predicting gross total stem volumes in cubic feet for six major conifers of southwest Oregon. Forest Research Laboratory, Oregon State University, Corvallis. Research Bulletin 50. 36 p.

WENSEL, L.C. 1977. A generalized prismoidal log volume equation. Department of Forestry and Conservation, University of California, Berkeley. Biometrics Note 5. 6 p.

Appendix A: Equations Predicting V_{abh}

Two equations predicting V_{abh} were developed by Walters et al. (1985). Equation [A.1] can be applied to all species. Equation [A.2] incorporates crown ratio as an additional variable and can be applied only to Douglas-fir and grand or white fir.

$$\hat{V}_{abh} = b_1 \, [H_{abh}/DOB]^{b_2} \, DOB^2 \cdot H_{abh} \quad [A.1]$$

$$\hat{V}_{abh} = c_1 \, [H_{abh}/DOB]^{c_2} \cdot EXP\,[c_3 \, CR_{abh}] \, DOB^2 \cdot H_{abh} \quad [A.2]$$

where:

V_{abh} = estimated volume from breast height to tree top
DOB = diameter outside bark at breast height
H_{abh} = total height above breast height
CR_{abh} = crown ratio for stem above breast height
= $(H - CB)/H_{abh}$ for CB > 4.5
= 1.0 for CB $\leq$ 4.5
CB = height to live crown base.

Coefficients b_1, b_2, c_1, c_2, and c_3 are presented in Table A–1.

TABLE A–1.

REGRESSION COEFFICIENTS FOR PREDICTING V_{abh}, EQUATIONS [A.1] and A.2].

Species	Regression coefficients for Equation [A.1]		Regression coefficients for Equation [A.2]		
	b_1	b_2	c_1	c_2	c_3
Douglas–fir	0.001168	0.265430	0.001420	0.211235	–0.173824
Grand/white fir	0.001080	0.358300	0.001625	0.233905	–0.302363
Ponderosa pine	0.001265	0.172813	NA	NA	NA
Sugar pine	0.000866	0.383940	NA	NA	NA
Incense–cedar	0.000887	0.367622	NA	NA	NA

Source: Walters et al. (1985).

Appendix B: Equations Predicting DIB

An equation was developed by Larsen and Hann (1985) to estimate DIB:

$$\widehat{DIB} = e_1 \, DOB^{e_2} \qquad [B.1]$$

where:

$\widehat{DIB}$ = estimated diameter inside bark at breast height

DOB = diameter outside bark at breast height.

Coefficients e_1 and e_2 are presented in Table B–1.

TABLE B–1.

REGRESSION COEFFICIENTS FOR PREDICTING DIB, EQUATION [B.1].

Species	Regression coefficients for Equation [B.1]	
	e_1	e_2
Douglas–fir	0.903563	0.989388
Grand/white fir	0.904973	1.000000
Ponderosa pine	0.809427	1.016866
Sugar pine	0.859045	1.000000
Incense–cedar	0.837291	1.000000

Source: Larsen and Hann (1985).

Appendix C: Equations Predicting $dib_{1.0}$

Walters et al. (1985) developed two equations estimating diameter inside bark at 1.0 foot above tree base ($dib_{1.0}$):

$$\widehat{dib}_{1.0} = f_0 + f_1 \ DOB^{f_2} \qquad [C.1]$$

$$\widehat{dib}_{1.0} = g_0 + g_1 \ EXP\ [g_2 \ CR] \ DOB^{g_3} \qquad [C.2]$$

where:

$\widehat{dib}_{1.0}$ = estimated diameter inside bark at 1.0 foot above tree base

H = total tree height
CB = height to live crown base
CR = crown ratio for the total stem = (H − CB)/H.

Coefficients f_0, f_1, f_2, g_0, g_1, g_2, and g_3 are presented in Table C-1. Equation [C.1] can be applied to all species, but Equation [C.2] is applicable only to Douglas-fir and grand or white fir.

TABLE C-1.

REGRESSION COEFFICIENTS FOR PREDICTING $dib_{1.0}$, EQUATIONS [C.1] and [C.2].

Species	Regression coefficients for Equation [C.1]			Regression coefficients for Equation [C.2]			
	f_0	f_1	f_2	g_1	g_2	g_2	g_3
Douglas-fir	0.000000	0.989819	1.000000	0.000000	0.938343	0.101792	1.000000
Grand/white fir	0.287414	0.828652	1.082631	0.341157	0.753147	0.101138	1.095299
Ponderosa pine	0.000000	1.000000	1.000000	NA	NA	NA	NA
Sugar pine	0.000000	1.039080	1.000000	NA	NA	NA	NA
Incense-cedar	0.476734	0.819613	1.067437	NA	NA	NA	NA

Source: Walters et al. (1985).

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