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Mortality Equations for Individual Trees in the Mixed-Conifer Zone of Southwest Oregon

David W. Hann
Chao-Huan Wang



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Acknowledgments

This study was conducted as part of the Forestry Intensified Research (FIR) Program, a cooperative effort of Oregon State University, the USDA Forest Service, and the USDI Bureau of Land Management. We thank Boise Cascade Corporation and Medford Corporation for their special assistance.

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Research Bulletin Oregon State University, Forest
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Abbreviations

BA	Total basal area of the stand
BAF	Basal area factor
BAL	Basal area in trees larger than the subject tree
CCFL	Crown competition factor in trees larger than the subject tree
CCH	Crown closure at the top of the tree
CR	Crown ratio
DBH	Diameter at breast height
EXPAN	Expansion factor (number of trees represented per acre)
H	Total tree height
HCB	Height to live crown base
PM	Probability of death during the next growth period for a given tree
PS	Probability of survival through the next growth period for a given tree
QMD	Quadratic mean diameter of the stand
RD	Relative density of the stand
SDI	Stand-density index
SI	Site index (Hann and Scrivani 1987)

Specific cases of these terms (e.g., DBH_1 and DBH_2), as well as additional terms used in the equations, are explained in the text.

Abstract

Models of stand growth and yield must include an estimate of mortality. For individual-tree/distance-independent growth-and-yield models, it is necessary (1) to predict the probability of death of individual trees and (2) to keep stand projections within reasonable biological limits (e.g., the maximum basal area that the stand can achieve).

Introduction

Because mortality is an important component of stand development, any model constructed to predict stand growth and yield must include prediction of mortality rates. The probability of mortality of a given tree frequently depends on its size and vigor, its position in the stand, and stand stocking or density (Hamilton 1986).

Many recent mortality models have used non-linear logistic equations to predict the probability of death of an individual tree during a specified growth period (Hamilton and Edwards 1976, Monserud 1976, Krumland *et al.* 1977, Ferrell 1980, Wykoff *et al.* 1982, Buchman *et al.* 1983, Dahms 1983, Hamilton 1986). However, if the data sets used to develop such equations come either from plots measured over short periods or from temporary plots, predicted mortality rates could be unreasonably high or low. Mortality rates over short periods can be influenced greatly by such factors as weather and pests, while mortality in temporary plots often must be postdated, which can increase errors in measurement of mortality rates. Underestimation of mortality rates is of particular concern, because use of low mortality estimates in long-term projections of stand development would cause unreasonably high estimates of stand basal area and volume. As a result, allowable cuts could be overestimated and the forest cut at a rate inconsistent with sustained yield. To prevent the prediction of unreasonably high densities, Wykoff (1986) and Johnson *et al.* (1986) added additional mortality functions that increased

In this study, the probability of death of an individual tree is predicted by a logistic equation with coefficients estimated for tree species growing in the mixed-conifer zone of southwest Oregon. Overall stand projections are controlled by use of size-density trajectories that govern the approach of the stand to the maximum size-density line.

the mortality rates if the stand approached an expected maximum density.

This report describes the development of two sets of equations. The first is used to predict the probability of individual-tree mortality for the following species found in southwest Oregon:

Douglas-fir	<i>Pseudotsuga menziesii</i> (Mirb.) Franco
Grand fir	<i>Abies grandis</i> (Dougl. ex D. Don) Lindl.
White fir	<i>Abies concolor</i> (Gord. & Glend.) Lindl. ex Hildebr.
Ponderosa pine	<i>Pinus ponderosa</i> Dougl. ex Laws.
Sugar pine	<i>Pinus lambertiana</i> Dougl.
Incense-cedar	<i>Calocedrus decurrens</i> Torr.
Madrone	<i>Arbutus menziesii</i> Pursh
Chinkapin	<i>Castanopsis chrysophylla</i> (Dougl.) A. DC.
Tanoak	<i>Lithocarpus densiflorus</i> (Hook. & Arn.) Rehd.
California black oak	<i>Quercus kelloggii</i> Newb.

The second set of equations is used to limit maximum densities to reasonable values. These equations are applied in the southwest Oregon version of ORGANON (Hester *et al.* 1989), an individual-tree/distance-independent model of growth and yield.

Background

Individual-Tree Mortality Equations

Growth-and-yield models based on individual trees characterize the stand through a statis-

tically selected sample of trees. An expansion factor associated with each sampled tree estimates the number of trees per acre that the tree represents. The total number of trees per acre in the stand is estimated by summing the expansion

factors for all sampled trees. Estimates of future number of trees in the stand are obtained by reducing the expansion factor of each sampled tree by its predicted mortality rate.

Two basic approaches have been used to model the mortality rate of individual trees. In one, the total number of trees that will die in the stand is predicted. This number is allocated to individual trees on the basis of diameter: trees smallest in diameter are assigned the greatest mortality rates (Campbell *et al.* 1979, Arney 1985, Johnson *et al.* 1986). The allocation is often based on the subjective judgment of the modeler or user, not on statistical analysis.

In the other approach, regression analysis is used to develop an equation that predicts the probability of the tree dying (or surviving) in the next growth period. While a few studies, such as Lanford and Cunia (1977), have used a linear model to predict the probability of mortality (PM) [or survival (PS)], the vast majority have chosen nonlinear models, the most popular of which has been the following generalized logistic:

$$PM \text{ or } PS = c_0 + \frac{1.0}{1.0 + \text{EXP}[f(\underline{X}, \underline{b})]} \quad [1]$$

where

PM = the probability of the tree dying in the next growth period,

PS = the probability of the tree surviving in the next growth period
= 1.0 - PM,

c_0 = regression parameter,

EXP(a) = e^a , where e is the base of the natural logarithm (ln),

\underline{X} = an array of independent variables,

\underline{b} = an array of additional regression parameters, and

$f(\underline{X}, \underline{b})$ = a linear or nonlinear function of the values in parentheses.

In the most widely used form of equation [1], c_0 is set to 0 and $f(\underline{X}, \underline{b})$ is expressed as a linear function (Hamilton and Edwards 1976, Monserud 1976, Krumland *et al.* 1977, Ferrell 1980, Wykoff *et al.* 1982, Hamilton 1986). Hamilton (1986) listed the following advantages of this formulation:

"The function is bounded by zero and one. Thus, the function is naturally limited to the potential range of probabilities of mortality.

"With the selection of the proper set of variables and transformations, the logistic function can be used to describe most naturally occurring patterns of mortality.

"Nonlinear estimation routines can efficiently estimate the parameters of the logistic function. This efficiency does not appear to be affected by the selection of starting values."

Dahms (1983) also set c_0 of equation [1] to 0, but he used a nonlinear expression for $f(\underline{X}, \underline{b})$. Buchman *et al.* (1983) incorporated c_0 into their equation to estimate the minimum background mortality that occurs regardless of the attributes of the tree or the stand. As a result, predictions from their model are bounded by c_0 and $1 + c_0$, rather than 0 and 1. Buchman *et al.* (1983) also used a nonlinear expression for $f(\underline{X}, \underline{b})$.

Although the general form used to model mortality has been quite similar in many studies, the particular independent variables have varied substantially from study to study. These variables can be classed into four groups characterizing tree size, tree vigor, tree position within the stand, or stand density.

The tree-size variable used most commonly has been diameter outside bark at breast height (DBH) or transformations of it (Hamilton and Edwards 1976, Monserud 1976, Krumland *et al.* 1977, Wykoff *et al.* 1982, Buchman *et al.* 1983, Hamilton 1986). Hamilton and Edwards (1976) also used total tree height in equations for four species. In general, the predicted effect of increasing tree size is decreased mortality rate. However, the equations of Buchman *et al.* (1983) predict an initial decrease, followed by an increase, in the rate of mortality as tree diameter increases, and those developed by Wykoff *et al.* (1982) predict an increased rate of mortality as tree diameter increases.

The basic tree attributes used to construct independent variables related to tree vigor have included actual diameter growth rate for the previous growth period (Buchman *et al.* 1983, Hamilton 1986), predicted diameter growth rate for the next growth period (Monserud 1976), and crown ratio (Ferrell 1980), defined as the crown length divided by total height. In all of these studies, predicted mortality rate decreases as diameter growth or crown ratio increases.

Independent variables related to tree position have included crown class of the tree (Hamilton and Edwards 1976, Ferrell 1980), the ratio of average basal area in the stand to basal area of the tree (Krumland *et al.* 1977), the ratio of diameter of the tree to average diameter in the stand (Hamilton 1986), and the ratio of the number of trees in the stand with diameter equal to or smaller than that of the subject tree to the total number of trees in the stand (Dahms 1983). The equations developed in all of these studies predict lower mortality rates in dominant trees than in trees in inferior positions in the stand.

Independent variables related to stand density have included stand basal area (Hamilton and Edwards 1976), the square root of stand basal area (Hamilton 1986), and stand density in trees per acre (Krumland *et al.* 1977). The first two studies predict increased rate of mortality as stand basal area increases, while the third study predicts decreased rate of mortality as stand density increases.

Maximum Density

In version 5.0 of the PROGNOSIS growth-and-yield model, Wykoff (1986) combined an equation for predicting mortality of individual trees with equations for controlling both individual-tree mortality rates and maximum stand density. The equation to predict mortality of individual trees was developed from long-term research data from permanent plots on three national forests in northern Idaho. This equation was then modified to (1) correct for differences in habitat types or national forest and (2) increase the mortality rate resulting from approach of stand basal area to the maximum for the habitat type. The correction equation for habitat type and national forest was

COR = correction for differences in habitat or national forest,

$$= \frac{PM_h}{PM_p}$$

where

PM_h = predicted mortality rate for the habitat type and national forest, based on stand-density index (SDI) (Reineke 1933)

$$= 1.0 - \left(\frac{QMD + PQMDG_h}{QMD} \right)^{-1.605}$$

PM_p = predicted mortality rate for the permanent-plot data set, based on SDI (Reineke 1933)

$$= 1.0 - \left(\frac{QMD + PQMDG_p}{QMD} \right)^{-1.605}$$

QMD = current quadratic mean diameter of the stand,

PQMDG_h = predicted potential diameter growth for the habitat type and national forest, and

PQMDG_p = predicted potential diameter growth for the permanent-plot data set.

The adjustment equation for approach to maximum basal area was

PM_b = predicted annual mortality rate resulting from approach of the stand to its maximum basal area

$$= 1.0 - \left[1.0 - \frac{(N - NB_{10})}{N} \right]^{0.1}$$

where

N = current number of trees per acre in the stand, and

NB₁₀ = predicted number of trees 10 years in the future, after mortality caused by approach to maximum basal area

$$= \frac{BA + \left(1 - \frac{BA}{BAMAX} \right) SBAG}{0.005454154(QMD + QMDG)^2}$$

where

BA = current stand basal area per acre,

BAMAX = maximum stand basal area per acre for the habitat,

SBAG = predicted gross basal area growth of the stand for the next 10 years,

QMD = current quadratic mean diameter of the stand, and

QMDG = predicted change in quadratic mean diameter over the next 10 years.

Wykoff combined these equations in the following manner:

$$PM = PM_b \left(\frac{BA}{BAMAX} \right) + \left(1 - \frac{BA}{BAMAX} \right) (PM_t)(COR)$$

where

PM = combined predicted probability of a tree dying,

PM_t = predicted annual individual tree mortality rate,

and other terms are as defined in the related equations, above.

For the SORNEC version of the PROGNOSIS model, Johnson *et al.* (1986) used the SDI concepts of Reineke (1933) to limit the maximum potential density of a stand. Their method first estimates maximum SDI for the stand, based on the most abundant species in the stand. If the actual SDI exceeds this estimate, the maximum is increased to the actual SDI value.

Two additional SDI values are then computed: one at 85 percent and the other at 55 percent of the maximum. It is assumed that

mortality rates will be low when SDI is below 55 percent of maximum and that SDI will not exceed 85 percent of maximum. If the starting SDI is above 85 percent of maximum, the mortality rate is increased in order to place the stand at this level at the end of the first growth period, and the subsequent mortality rates are determined so as to maintain the SDI of the stand at 85 percent of maximum for all future growth periods. If the SDI falls between 55 percent and 85 percent of maximum, the mortality rates for subsequent growth periods are increased, but the SDI values also are increased until they reach 85 percent of maximum, after which SDI remains at this level. The rate at which the stand approaches the 85 percent level depends upon the QMD of the stand; stands with small QMDs approach faster than larger ones. If the SDI is below 55 percent of maximum, the mortality rate is set at a small "background" value.

Data Description

Two sources of data were used in this analysis. Individual-tree mortality equations were developed from data collected in temporary plots. Data from permanent plots were used to develop a maximum size-density line and a size-density trajectory to control the maximum basal area of the stand.

Data from Temporary Plots

Data Collection

The study area for the first data set extends from near the California border (42°10'N) on the south to Cow Creek (43°00'N) on the north and from the Cascade crest (122°15'W) on the east to approximately 15 miles west of Glendale (123°50'W) (Figure 1). Elevation ranges from 900 to 5100 feet. Mean minimum temperature in January ranges from 23 to 32°F, and mean maximum temperature in July, from 79 to 90°F. Annual precipitation varies from 29 to 83 inches; less than 10 percent falls during June, July, and August.

The data were collected during 1981, 1982, and 1983 from temporary plots established in 391 stands as part of the Growth-and-Yield Project of the Forest Intensified Research (FIR) Program. In each stand, a cluster of 4 to 10 variable-radius plots and 2 nested, fixed-area subplots was estab-

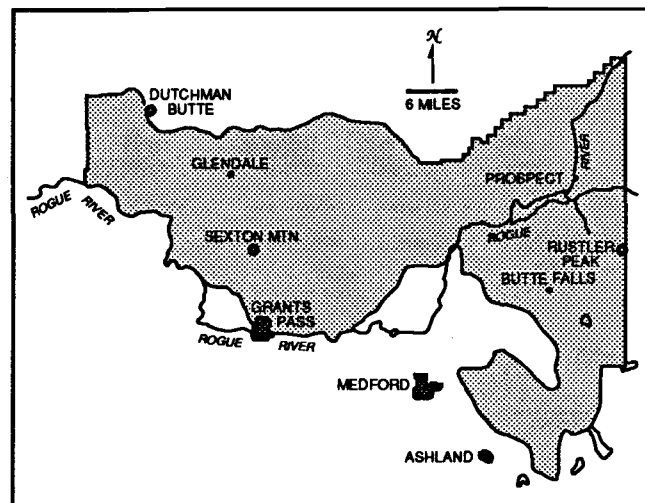


Figure 1. Map of the study area (shaded) in which the temporary plots were measured.

lished. A variable-radius plot with a basal area factor (BAF) of 20 was used for trees with DBH ≥ 8.1 inches; a circular fixed-area subplot with a radius of 15.56 feet was used for trees with $4.1 \leq \text{DBH} \leq 8.0$ inches; and a circular fixed-area subplot with a radius of 7.78 feet was used for trees with DBH ≤ 4.0 inches.

The following measurements were taken for all trees more than 6 inches tall: a mortality indicator of whether the tree died during the past

5 years, DBH (DBH_2), total tree height (H_2), height to live crown base (HCB_2), and horizontal distance from plot center to tree center (DIST). In addition, radial and height growth during the past 5 years were measured on subsamples.

Date of death was based on physical features (USDA Forest Service 1978, Cline *et al.* 1980). DBH was measured to the nearest 0.1 inch with a diameter tape. H_2 and HCB_2 were determined by the tangent method (Curtis and Bruce 1968, Larsen *et al.* 1987); position of the crown base was determined by visual reconstruction, in which gaps in the crown were filled in with branches from below the crown base to produce a symmetric, even-based crown.

The distance from plot center to tree center was determined by adding one-half DBH_2 to the horizontal distance from plot center to tree face. Past radial growth at breast height was measured with an increment borer on all trees with large enough DBH. Five-year height of all undamaged trees under 25 feet tall was measured with a 25-foot telescoping pole. For trees taller than 25 feet, a subsample of up to six trees on each plot were felled and sectioned at the first and sixth whorls. The ages at these whorls were determined to ensure a true 5-year growth period, and the distance between the whorls was measured.

Definition and Derivation of Variables

The expansion factor ($EXPAN_2$), or number of trees represented per acre, for a sampled tree alive

at the end of the growth period was assigned by the following rules:

1. $EXPAN_2 = 229.18$ trees per acre for trees with $DBH_2 \leq 4.0$ inches.
2. $EXPAN_2 = 57.30$ trees per acre for trees with $4.0 < DBH_2 \leq 8.0$ inches.
3. $EXPAN_2 = 3666.93 (DBH_2)^2$ for trees with $DBH_2 > 8.0$ inches.

Because the objective of the project was to estimate future, rather than past, mortality rates, it was necessary to estimate the values of all tree measurements at the start of the previous 5 years (DBH_1 , H_1 , HCB_1 , and $EXPAN_1$). The procedures used to backdate the tree measurements are described in detail in the Appendix. If $EXPAN_1$ was 0, the tree was excluded from the analysis.

After the basic tree variables had been backdated, several tree, tree-position, and stand variables were calculated. The variables that were used in the final individual-tree mortality equations are summarized in Table 1.

A dichotomous mortality variable was formed for each tree, with a value of 1 if the tree died in the next 5-year growth period and a value of 0 if it did not. Crown ratio at the start of the growth period (CR_1) was determined by

$$CR_1 = 1.0 - \frac{HCB_1}{H_1}$$

where HCB_1 is height to live crown base and H_1 is total tree height, both at the start of the 5 years.

Table 1. Summary of variables used in developing final individual-tree mortality equations. CCH, crown closure at top; BAL, basal area for trees larger than the subject tree; DBH, diameter outside bark at breast height; CR, crown ratio; SI, site index (Hann and Scivani 1987).

Species	Number of plots	Number of trees	CCH		BAL (ft ²)		DBH (in.)		CR		SI (ft)	
			Mean	Range	Mean	Range	Mean	Range	Mean	Range	Mean	Range
Douglas-fir	389	13336	20.5	0.0-174.1	101.9	0.0-380.0	12.2	0.1-83.8	0.48	0.00-1.00	93.1	54.1-141.1
Grand and white fir	328	2357	19.9	0.0-172.8	111.6	0.0-387.2	11.7	0.1-51.1	0.52	0.01-1.00	92.7	59.4-141.1
Ponderosa pine	154	1566	12.5	0.0-142.7	80.6	0.0-272.1	14.4	0.1-59.6	0.44	0.02-1.00	88.5	54.8-141.1
Incense-cedar	178	1433	47.2	0.0-147.5	127.1	0.0-322.4	7.1	0.1-66.3	0.49	0.03-1.00	88.3	54.8-124.3
Sugar pine	144	364	12.5	0.0-147.1	56.8	0.0-265.8	16.8	0.1-59.9	0.51	0.10-1.00	86.7	54.1-126.6
Madrone	176	1287	27.0	0.0-122.8	98.8	0.0-303.2	7.0	0.1-44.5	0.44	0.00-1.00	96.6	54.1-142.0
Chinkapin	113	952	52.7	0.0-142.9	109.0	0.0-302.0	3.0	0.1-26.5	0.49	0.01-1.00	90.0	54.1-130.6
Tanoak	43	400	80.5	3.5-142.9	127.2	1.5-283.9	1.2	0.1-11.8	0.57	0.06-1.00	88.9	54.1-126.6
California black oak	57	358	36.4	0.0-134.3	101.2	0.0-294.4	11.1	0.1-48.4	0.41	0.02-1.00	84.6	54.8-125.6

Three variables—basal area in larger trees (BAL_1), crown competition factor in larger trees ($CCFL_1$), and crown closure at the top of the tree (CCH_1)—were used to quantify the position of a tree in the stand at the start of the growth period.

BAL_1 is the sum of the basal areas of trees in the stand with DBH_1 larger than that of the subject tree. Therefore, the BAL_1 of the tree with the largest diameter is 0, whereas that of the tree with the smallest diameter is slightly less than the total basal area of the stand. Similarly, $CCFL_1$ is the crown competition factor (CCF) in trees with DBH_1 larger than that of the subject tree. The CCF is the ratio of the sum of the maximum crown areas for all trees of interest in the stand or plot to the area of the stand or plot (Krajicek *et al.* 1961). This ratio is multiplied by 100 and expressed as a percent. Maximum crown areas were computed from equations for maximum crown width developed for trees in southwest Oregon (Paine and Hann 1982).

To calculate CCH_1 of a particular tree, H_1 was used to define a reference height. Crown widths at the reference height for all other trees in the stand were estimated with the crown-width equations in Ritchie and Hann (1985). If the reference height fell above the top of a tree, crown width was 0; if it fell below crown base of a tree, crown width at crown base was used. Crown width was converted to crown area by the formula for the area of a circle. The crown areas were summed and expressed as a percentage of acreage covered. This procedure was repeated for all trees in the stand.

Variables calculated for the stand included total basal area at the start of the growth period (BA_1), total crown competition factor at the start of the growth period (CCF_1), and site index (SI) from Hann and Scrivani (1987).

Data from Permanent Plots

The sources of the second data set were permanent plots on 24 research installations in southwest Oregon. These installations were part of a set used by Miller *et al.* (1988) to develop equations for gross volume growth and response to thinning and fertilizing of Douglas-fir stands. All installations near the study area of the first data set were visited by people involved in collecting that data. Installations with stand structures, species mixes, and soil types similar to the stands measured in the first data set were included in the second. Each installation selected contained 4 to 18 fixed-area plots, with sizes ranging from 0.1 to 0.2 acre. A total of 128 plots were chosen: 45 controls, 48 fertilized plots, 24 thinned plots, and 11 thinned and fertilized plots. The plots had been remeasured every 2 to 5 years; the total time over which measurements had been made ranged from 4 to 15 years, with an average of 8.5 years. At each remeasurement, all DBHs greater than 1.5 inches were measured to the nearest 0.1 inch. Total tree heights were measured to the nearest foot on a subsample of 10 to 20 trees per plot. Crown ratios were measured on a subsample in one installation.

Data Analysis

Because the data used to develop the other components of the ORGANON growth-and-yield model (Hester *et al.* 1989) came from temporary plots, we developed individual-tree mortality equations from the temporary-plot data and then used the permanent-plot data to develop a separate function for limiting stand basal area to reasonable values for southwest Oregon.

Mortality Equations Based on Individual Trees

The probability of a tree dying in a 5-year growth period was modeled by equation [1], with c_0

set to 0 and $f(\underline{X}, \underline{b})$ expressed as a linear function. The regression coefficients \underline{b} can be estimated by either weighted least squares or maximum likelihood estimation (Hamilton 1986). For this study, we estimated the parameters with the weighted least squares regression package RISK (Hamilton 1974). The dependent variable used to estimate the probability of mortality, PM, was the dichotomous mortality variable. The independent variables to be included were determined by the following process.

1. The independent variables were classified into five groups, based on the attributes they represent:

- a. DBH_1 (tree size)
 - b. CR_1 (tree vigor)
 - c. BAL_1 , $CCFL_1$, and CCH_1 (tree position in the stand)
 - d. BA_1 and CCF_1 (stand density)
 - e. SI (productivity).
2. Many alternative sets of independent variables were formed by selecting at most one variable from each group.

provide reasonable comparisons of mortality rates among species was found.

6. The RISK fits to the data indicated that parameters for Douglas-fir and white and grand firs were similar for all independent variables except the intercept term. Therefore, the two data sets were combined and fitted with a species-group indicator variable.

The following equation form resulted from screening the independent variables:

$$PM = \frac{1}{1 + \text{EXP}[-(b_0 + b_1 DBH_1 + b_2 CR_1 + b_3 SI + b_4 BAL_1 + b_5 CCH_1)]} \quad [2]$$

3. Each of these sets was fit to each species with RISK.
4. The resulting parameters and statistics were examined in detail in order to determine if the parameters were significant ($p = 0.05$) by both of the t -tests in RISK and whether the signs of the parameters agreed with those in prior studies. Variables that were insignificant by either t -test were dropped from the equation. All equations with significant and reasonably signed parameters were ranked by chi-square goodness-of-fit statistics; the equation with the lowest chi-square statistic for each species was tested further in the ORGANON growth-and-yield model (Hester *et al.* 1989).
5. We used ORGANON to examine the pattern and rates of mortality for the various species in long-term projections for 42 plots. Because a large data set was available for Douglas-fir, it was used as the basis for comparison. If we judged the mortality rate of other species to be too high or low relative to Douglas-fir, alternative equations with higher chi-square statistics were tried until a set that appeared to

where

PM = probability of death in the next 5 years for a given tree,

EXP(a) = e^a , where e is the base of the natural logarithm (ln),

DBH_1 = diameter outside bark at breast height at the start of the growth period,

CR_1 = crown ratio at the start of the growth period,

SI = site index for the stand (Hann and Scrivani 1987),

BAL_1 = basal area in trees with DBH_1 larger than that of the subject tree at the start of the growth period,

CCH_1 = crown closure at the tip of the tree at the start of the growth period, and

$b_0 \dots b_5$ = regression coefficients (Table 2).

In no case were all variables in equation [2] included in a species-specific equation. For several species, one or more of the variables was not significant ($p=0.05$). In addition, only one of the independent variables related to tree position (BAL_1 and CCH_1) was used in any species-specific equation.

Table 2. Estimated regression coefficients for equations [2] and [5].

Species	b_0	b_1	b_2	b_3	b_4	b_5
Douglas-fir	-0.149558	-0.203923	-7.32001	0.0133533	0.00168508	0.0
White/grand fir	-1.00059	-0.203923	-7.32001	0.0133533	0.00168508	0.0
Ponderosa pine	-0.197353	-0.038126	-7.02697	0.0	0.0	0.011877
Incense-cedar	-1.05287	0.0	-7.97558	0.0	0.0047572	0.0
Sugar pine	0.230564	-0.090354	-2.62754	0.0	0.0	0.0
Madrone	-1.27314	-0.130403	-2.90376	0.0	0.0043943	0.0
Chinkapin	-0.405173	0.0	-6.85595	0.0	0.0050301	0.0
Tanoak	-0.203167	0.0	-6.47286	0.0	0.00774629	0.0
California black oak	-6.54885	0.0	0.0	0.0	0.0217083	0.0

Maximum Density

We felt that the approaches used by Wykoff (1986) and Johnson *et al.* (1986) were either too complex or too subjective. As an alternative, we used maximum size-density concepts to limit the maximum potential density of a stand. These concepts have been widely developed and used. For example, Johnson *et al.* (1986) based their method for limiting maximum density on the maximum SDI ideas of Reineke (1933).

In general, maximum size-density concepts are based on the observation that stands approach a limit over time that defines maximum average size per tree in stands of a given density (Reineke 1933, Yoda *et al.* 1963, Drew and Flewelling 1977, 1979). This limit has often been characterized by the following maximum size-density line:

$$MY_1 = a_1 + a_2 X_1, \quad [3]$$

where

MY_1 = natural log of maximum average size per tree for a given number of trees,

X_1 = natural log of number of trees per acre, and

a_1, a_2 = parameters.

More recently, Smith and Hann (1984, 1986) developed the following equation to characterize the approach of a stand to its maximum size-density line:

$$Y_1 = MY_1 - (a_1 + a_2 X_0 - Y_0) \text{EXP} \left[-a_3 (X_0 - X_1)^{a_4} \right], \quad [4]$$

where

Y_1 = natural log of average size per tree at time i ,

X_0 = natural log of initial number of trees in the stand when mortality starts,

Y_0 = natural log of average size per tree at the beginning of mortality,

a_3, a_4 = additional parameters,

and other terms are as defined in the preceding equations.

Smith and Hann (1984, 1986) also demonstrated how to estimate the parameters of equation [4] by using nonlinear regression techniques and long-term, repeated measurements from permanent plots.

We estimated the parameters of equation [4] from the data from the control and fertilization plots of the permanent-plot data set. The data from the fertilized plots were included because the size-density relationship has been found to be independent of productivity (White and Harper 1970, Smith and Hann 1984). For mean tree size, we chose the QMD of the stand, because it is easily calculated from stand basal area and number of trees and has been widely used (Reineke 1933, Johnson *et al.* 1986, Hyink *et al.* 1988).

A preliminary fit of the data to equation [4] produced parameter estimates for a_3 and a_4 that were not significantly different ($p = 0.01$) from 0. The intercept value (a_1) of the resulting maximum size-density line was lower and the slope was less steep than those obtained by Reineke (1933). We attribute these problems to the relatively short-term measurements from permanent plots and to the small number of installations available. The equation is probably overparameterized for the data available. We therefore simplified equation [4] by fixing as many parameters as possible to values we judged reasonable. We concluded from examining the values found by Smith and Hann (1984) that a_4 could be safely set to 1.0. We next set a_2 to Reineke's value of -0.62305, which has been found to be appropriate for a wide range of species (Reineke 1933, Johnson *et al.* 1986, Hyink *et al.* 1988). We estimated a_1 and a_3 using nonlinear regression. The resulting value for a_1 was 6.21113, which produces a maximum SDI of 530.2. Reineke's maximum SDI for Douglas-fir was 595, based on an intercept of 6.3. Therefore, our maximum size-density line is 89.11 percent of Reineke's maximum size-density line. The final value for a_3 was 1.47343. The adjusted coefficient of determination (R^2) for the equation was 0.9243. Both parameters were significantly different from 0 ($p = 0.01$).

Reineke (1933) also found that the intercept of the size-density line depended on the species of the stand. We did not have data to develop separate size-density trajectories directly for other species in southwest Oregon. Therefore, we used the following procedure to develop correction factors to convert the intercept value for Douglas-fir to intercept values for other species.

Johnson *et al.* (1986) present two sets of species-specific maximum SDIs obtained from the timber management staffs of Regions 5 (California) and 6 (Oregon and Washington) of the U.S.

Forest Service. We converted each set for Douglas-fir, white fir and ponderosa pine to intercept parameters of equation [3] by the relationship

$$a_1 = \ln(10) + 0.62305[\ln(SDI_m)]$$

where

- a_1 = the intercept parameter of equation [3],
- $\ln()$ = the natural log of the value in parentheses, and
- SDI_m = the maximum stand-density index for the species.

For each data set, the intercept parameters for white fir and ponderosa pine were divided by that for Douglas-fir; these ratios were averaged across the two data sets to obtain a correction factor of 1.03482 for white fir and 0.99436 for ponderosa pine (Table 3). We estimated the intercept parameter for a pure stand of white and grand fir in southwest Oregon by multiplying the intercept parameter for Douglas-fir (6.2113) by the correction factor for white fir (1.03482). The resulting value (6.42852) gives a maximum SDI of 751.6 for mixed white and grand fir in southwest Oregon. The intercept parameter for a pure ponderosa pine stand in southwest Oregon was computed as 6.17386, with a corresponding maximum SDI of 499.4.

Table 3. Maximum values of stand density index (SDI), corresponding intercept terms (a_1) for equation [3], and resulting correction factors for Douglas-fir, ponderosa pine and white fir.

Species	U.S.F.S. Region 5 ¹	U.S.F.S. Region 6 ¹	Average
Maximum SDI values			
Douglas-fir	737	447	
Ponderosa pine	685	429	
White fir	1004	659	
Values of a_1			
Douglas-fir	6.41633	6.10478	
Ponderosa pine	6.37074	6.07918	
White fir	6.60895	6.34663	
Correction factors ²			
Douglas-fir	1.0	1.0	1.0
Ponderosa pine	0.99290	0.99581	0.99436
White fir	1.03002	1.03962	1.03482

¹ Region 5 includes California; Region 6 covers Oregon and Washington.

² Correction factors are computed by dividing a_1 for a given species by a_1 for Douglas-fir.

We recommend the following method to obtain an intercept term for a mixed-species stand. First, the initial stand basal areas in Douglas-fir, white and grand fir, and ponderosa pine are computed and expressed as a percentage of the sum of the three values. If the resulting percentage in Douglas-fir is ≥ 50 percent, the stand is assumed to be dominated by Douglas-fir. The stand is assumed to be dominated by white and grand fir or by ponderosa pine if the basal area in the given species is ≥ 67 percent. (Because Douglas-fir grows faster than these firs or ponderosa pine in southwest Oregon, we believe a higher percentage of the latter species is necessary to guarantee their domination of the site.) For stands that do not meet these requirements, the intercept term is computed as a weighted average of the three species. For example, the intercept term for a stand with 1/3 of its basal area in each of the three tree types would be $6.27117 [1/3 (6.21113) + 1/3 (6.42852) + 1/3 (6.17386)]$.

The final values needed to complete the size-density trajectory are X_0 and Y_0 , the values of X and Y when self-thinning mortality starts. Drew and Flewelling (1979) hypothesized that X_0 and Y_0 fall on a line paralleling the maximum size-density line. If the lines are assumed to be parallel, the problem is simplified to that of finding the relative density (RD) at which self-thinning begins to occur, where RD is defined as

$$RD = \frac{SDI}{SDI_m}$$

where

SDI = stand density

$$= N \left(\frac{10.0}{QMD} \right)^{a_2}$$

N = total number of trees per acre in the stand,

QMD = quadratic mean diameter of the stand, and

SDI_m = the maximum stand-density index for the species.

Long (1985) estimated that self-thinning would start at a RD of 0.6 for stands of Douglas-fir, ponderosa pine, western hemlock, and lodgepole pine. To check this value, we computed the annual mortality rate in the control plots of the perma-

ment-plot data set (expressed as a percent of the trees alive at the start of the growth period) and the RD at the start of the growth period for each growth period. Mortality data were divided into four classes on the basis of relative density and averaged within each class (Table 4). Below a RD of 0.6, the average annual mortality rate was 1.25 percent; the rate almost doubled, to 2.32 percent, in the 0.6 to 0.6999 range of RD. Because of the small sample size and short duration of measurements, a definitive conclusion could not be drawn from these results. However, we felt that the data does not refute the hypothesis that self-thinning starts at $RD = 0.6$; we therefore accepted this value for the stands of southwest Oregon.

Table 4. Average annual mortality rates, based on number of trees alive at start of the growth period, for the control plots of the permanent-plot data set.

Relative density at start of growth period	Number of plots	Average annual mortality rate (percent)
0.25 - 0.4999	10	1.08
0.50 - 0.5999	11	1.42
0.60 - 0.6999	18	2.32
0.70 - 0.7999	20	1.90

The values of X_0 , Y_0 , and the variables used to compute X_0 (described below) for a particular stand are determined at either the end of the growth period that first places the RD at or above 0.6, or the start of the first growth period, if the initial RD exceeds 0.6. The value of X_0 is computed by

$$X_0 = X_f + \left(\frac{1.0}{a_3}\right) \ln \left[\frac{\ln(0.6)}{\ln(RD_f)} \right],$$

where

X_f = natural log of the number of trees per acre in the stand,

a_3 = additional parameter, as in Equation [4], and

RD_f = relative density of the stand.

ship was derived under the assumption that X_f and Y_f (the natural log of the QMD associated with X_f) fall exactly on the size-density trajectory of the stand.

Given X_0 , Y_0 is computed by

$$Y_0 = 5.892866 - 0.62305(X_0),$$

which is the line parallel to the maximum size-density line that defines the start of self-thinning (i.e., $RD = 0.6$).

Combining Equations for Individual-Tree Mortality Rates with the Maximum-Density Function

The individual-tree mortality-rate equations can be combined with the self-thinning function by the following approach:

1. The individual-tree mortality rates are first computed by equation [2].
2. These mortality rates are used to compute the number of trees and QMD of the stand at the end of the growth period.
3. If the resulting QMD is less than or equal to the QMD predicted from the size-density trajectory of the stand (equation [4]), nothing further should be done.
4. If the QMD of the stand at the end of the growth period is greater than the QMD predicted from the size-density trajectory of the stand (equation [4]), mortality rates from equation [2] should be increased as necessary to restore the stand to the size-density trajectory.

The combined mortality rate can be expressed as

$$CPM = \frac{1}{1 + \text{EXP}[-KR(b_0 + b_1 DBH_1 + b_2 CR_1 + b_3 SI + b_4 BAL_1 + b_5 CCH_1)]} \quad [5]$$

Both X_f and RD_f are determined either at the end of the growth period that first places the RD at or above 0.6, or at the start of the first growth period, if the initial RD is above 0.6. This relation-

where

CPM = the combined probability of a tree dying in the next five years,

KR = coefficient to correct equation [2] and place stand on the size-density trajectory,

and other terms are as defined in equation [2].

The value of KR is determined by the RD of the stand at the start of the growth period and by the gross 5-year basal area growth of the stand. If RD at the start of the growth period is less than 0.6, KR should be set to 1.0, and mortality rate is predicted from equation [2]. If initial RD is between 0.6 and 1.0, different values for KR should be substituted in equation [5] until the number of trees and QMD fall on the size-density trajectory defined by equa-

tion [4]. If the starting RD is greater than 1.0, equation [5] should be solved iteratively with various values of KR until number of trees and QMD at the end of the growth period are adjusted to fall on the maximum size-density line defined by equation [3]. Finally, if the stand is thinned at the start of the growth period and RD at that time is greater than 0.6, KR should be set to 1.0 for all subsequent growth periods until the thinned stand again equals or exceeds the size-density trajectory of the unthinned stand, at which time equation [5] should again be solved iteratively in order to find a KR value that will keep the stand on the size-density trajectory.

Results and Discussion

The signs on the coefficients of equation [2] (Table 2) indicate the responses of mortality rate to changes in the predictor variables. Thus, the rate of mortality decreases as DBH or CR increases and increases as SI, BAL, or CCH increases.

Interactions of the independent variables in predicting the probability of mortality are illustrated in Figures 2 and 3. The curves in Figure 2 depict the lowest mortality rates possible for a given DBH and SI of Douglas-fir. The effect of increasing BAL or reducing CR on the predicted 5-year mortality rate of Douglas-fir growing on land with SI = 100 is illustrated in Figure 3 for trees with DBH of 5, 10, and 15 inches. These curves demonstrate that DBH and CR exert the most influence on predicted mortality rate of Douglas-fir, and the risk of dying is very low for trees with large diameters (DBH > 20 inches) or long crowns (CR > 0.7).

The X and Y value for each plot in the temporary plot and the permanent plot data sets, Reineke's maximum size-density line (SDI = 595), the maximum size-density line determined for southwest Oregon (SDI = 530.2), and the size-density line defining the start of self-thinning (SDI = 318.1) appear in Figure 4. The general overlap of the two data sets indicates that both should approach the same maximum size-density line. Figure 4 also shows that most of the stands were in the zone of self-thinning.

Examples of trajectories of stands approaching the southwest Oregon maximum size-density line are shown in Figure 5. Stands below the self-thinning line would follow a vertical or near-verti-

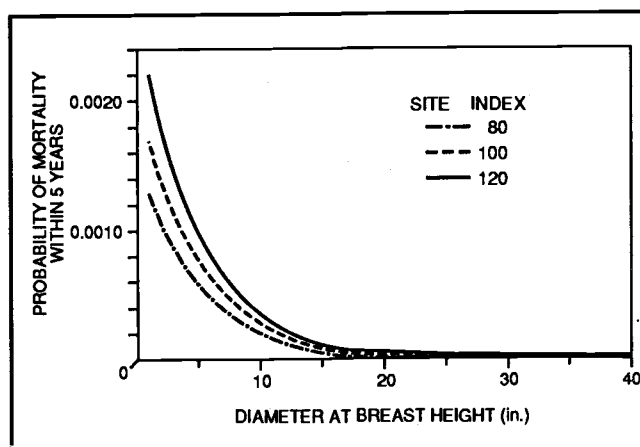


Figure 2. Predicted probability of mortality within 5 years for Douglas-fir trees with crown ratio (CR) = 1 and basal area of larger trees (BAL) = 0, when site index (SI) is 80, 100 or 120.

cal path to the self-thinning line (i.e., they should experience little or no mortality). Once on a trajectory, they move from right to left as they approach the maximum size-density line.

The shape of the size-density trajectory is independent of SI or age of the stand. For a given initial stand density, a stand with a higher SI moves along the trajectory faster than a stand with lower SI and therefore has a higher cumulative mortality rate. In general, thinning shifts the stand to the left on the size-density graph and decreases the mortality rate.

Size-density trajectories predicted from equation [4], such as the examples in Figure 5, represent the "average" path that stands follow as they

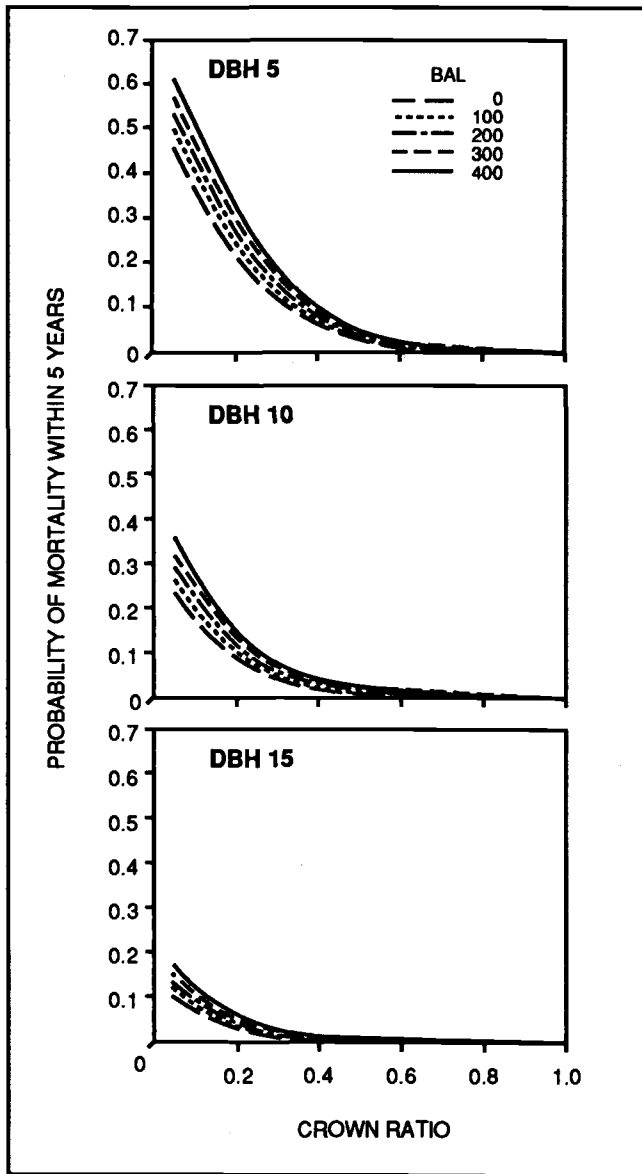


Figure 3. Predicted probability of mortality within 5 years for Douglas-fir trees when site index (SI) = 100, basal area of larger trees (BAL) ranges from 0 to 400 ft², and DBH = 5 inches, 10 inches, or 15 inches.

develop. Therefore, the maximum size-density line predicted from equation [4] represents the "average" maximum size-density line for the data set: consequently, individual stands that approached a maximum size-density line differing over time from that predicted from equation [4] would not be unusual. Deviation of a stand from the "average" size-density trajectory often occurs in response to erratic patterns of mortality caused by unusual climatic events or pest attacks.

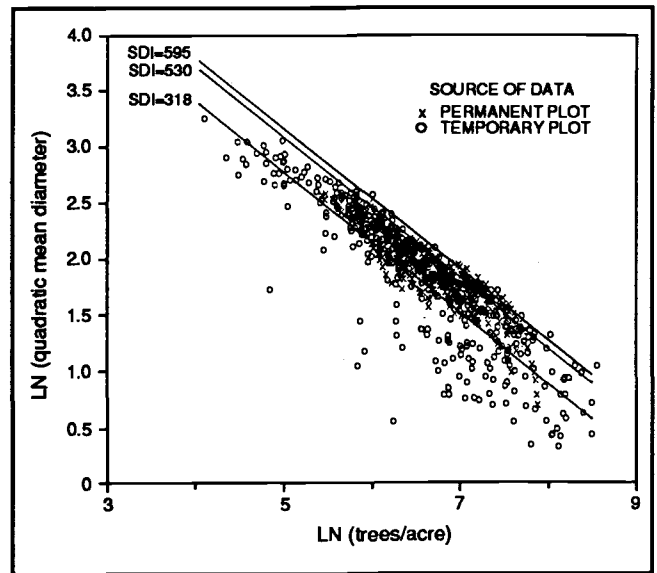


Figure 4. Size-density relationships for stands in southwestern Oregon. Data points marked with circles came from temporary plots located in 391 stands; data points marked with crosses came from 93 permanent plots located on 24 research installations. The upper line represents Reineke's (1933) maximum size-density line for Douglas-fir in California. The middle line is the maximum size-density line for Douglas-fir in southwestern Oregon, determined in this study. The lowest line is the size-density line at which "self-thinning" begins for Douglas-fir in southwest Oregon.

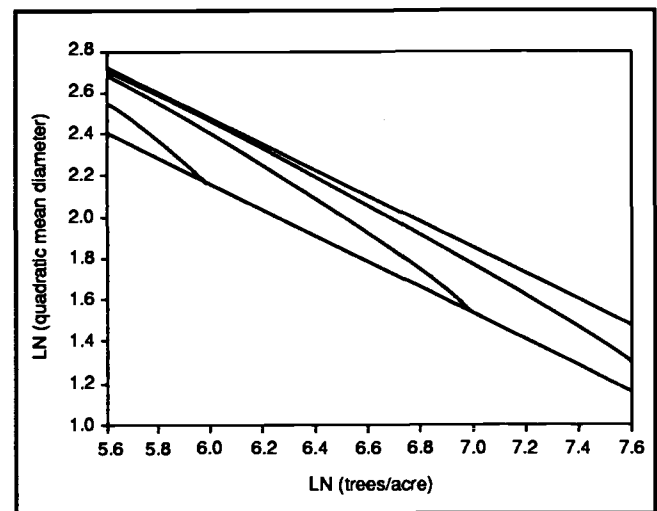


Figure 5. The size-density trajectory for Douglas-fir in southwestern Oregon. The upper straight line is the maximum size-density line and the lower straight line is the size-density line in which "self-thinning" begins. The three curved lines are examples of trajectories followed by stands while moving from the lower to the upper straight lines.

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Appendix: Backdating Tree Measurements

The following procedures were applied to the tree measurements from each stand to backdate them to the start of the previous 5-year growth period and permit conversion of past 5-year mortality rates into values for the future 5-year growth period.

DBH

The DBH at the start of the growth period (DBH_1) was estimated from the equation

$$DBH_1 = \left[\frac{A_1 (DBH_2^{A_2}) - 2 RG}{A_1} \right]^{\frac{1.0}{A_2}}$$

where

DBH_1 = DBH at the start of the growth period,

DBH_2 = DBH at the end of the growth period (i.e., measured DBH),

RG = measured 5-year radial growth of the tree, inside bark,

A_1, A_2 = regression coefficients (Larsen and Hann 1985) for predicting DBH inside bark from DBH outside bark.

On each plot, the data for each tree species with radial growths measured on at least five trees were used to develop the following regression equation:

$$\ln(\text{TBAG}_{\text{ib}}) = B_{0,1} + B_{1,1}(\text{DBH}_2)^2 + \ln(\text{DBH}_2), \quad [\text{A1}]$$

where

TBAG_{ib} = basal area growth inside bark

$$= \pi(\text{RG}) \left[A_1(\text{DBH}_2)^{A_2} - \text{RG} \right],$$

$\pi = 3.14159$,

RG = measured 5-year radial growth of the tree, inside bark,

DBH_2 = DBH at the end of the growth period (i.e., measured DBH),

A_1, A_2 = regression coefficients (Larsen and Hann 1985) for predicting DBH inside bark from DBH outside bark, and

$B_{0,1}, B_{1,1}$ = regression coefficients.¹

This equation is a simplified version of a form that has been successfully used by Wykoff *et al.* (1982), Ritchie and Hann (1985), Johnson *et al.* (1986) and Wykoff (1986) to predict basal area growth of individual trees. For species in which radial growths had been measured in fewer than five trees, species with similar growth forms were combined until at least five observations were available to estimate the parameters.

Equation [A1] was used to estimate DBH_1 for trees in the stand without a measured radial growth by applying the following relationship:

$$\text{DBH}_1 = \left[(\text{DBH}_2)^{2A_2} - \frac{4.0(\text{TBAG}_{\text{ib}})}{\pi(A_1)^2} \right]^{\frac{1}{2A_2}}$$

¹ In regression coefficients of the form $B_{i,j}$, i indexes regression coefficients within the equation and j indexes regression coefficients between equations.

where DBH_1 is DBH at the start of growth period, and all other terms are as defined in equation [A1].

Total Tree Height

For trees with a measured 5-year height-growth rate, total tree height at the start of the growth period (H_1) was determined by

$$H_1 = H_2 - \text{HG},$$

where

H_2 = total tree height at the end of the growth period, and

HG = 5-year height-growth rate.

For each stand, one or two equations were developed for each species to predict H_1 for trees without a measured 5-year height-growth rate. The number and type of equations developed for a particular species depended on the number of trees on the plot with total tree heights ≥ 20 feet, the number of trees with total heights < 20 feet, and the number of trees < 20 feet tall with measured 5-year height growths. We used two sets of equations, depending upon tree height, for two reasons.

1. We felt that developing two sets of equations would increase the precision of backdating the heights of shorter trees.
2. Height growths of trees < 25 feet tall were measured directly with the telescoping pole; as a result, more height-growth measurements were taken of the shorter trees. This opened the possibility of developing a stand-specific height-growth equation.

If six or more trees of a given species in a stand each had a total height ≥ 20 feet, the following height-diameter equation was fit to the data by least squares regression:

$$\ln(H_2 - 4.5) = B_{0,2} + B_{1,2} \left(\frac{1.0}{\text{DBH}_2} \right) \quad [\text{A2}]$$

This equation is very similar to that used by Wykoff *et al.* (1982). Total height at the start of the growth period (H_1) was estimated for all trees of the same species without measured 5-year height growths that were ≥ 20 feet high by equation [A2] and the relationship

$$H_1 = H_2 \left(\frac{PH1_1}{PH1_2} \right)$$

where

PH1₁ = predicted height at the start of the growth period, from the exponential of equation [A2]

$$= 4.5 + \text{EXP} \left[B_{0,2} + B_{1,2} \left(\frac{1.0}{DBH_1} \right) \right], \text{ and}$$

PH1₂ = predicted height at the end of the growth period, from the exponential of equation [A2]

$$= 4.5 + \text{EXP} \left[B_{0,2} + B_{1,2} \left(\frac{1.0}{DBH_2} \right) \right].$$

If a measured 5-year height growth was available for at least six trees of a given species < 20 feet tall, the following height-growth equation was developed by least squares regression:

$$\ln(HG) = B_{0,3} + B_{1,3} (\ln H_2), \quad [A3]$$

where

HG = 5-year height growth, and

H₂ = total height at the end of the growth period.

Wykoff *et al.* (1982) used equation [A3] to characterize height-growth rates of trees with DBH < 3 inches.

For trees < 20 feet tall without a 5-year height-growth measurement, we estimated H₁ by

$$H_1 = H_2 - \text{EXP} [B_{0,3} + B_{1,3} (\ln H_2)], \quad [A4]$$

If measured 5-year height growths were not available for at least six trees < 20 feet tall, but at least six trees of the given species were < 20 feet tall, the following allometric equation was fit to the data by least squares regression:

$$\ln(H_2 - 4.5) = B_{0,4} + B_{1,4} (\ln DBH_2). \quad [A5]$$

We estimated H₁ for all trees of the same species that were < 20 feet tall and on which 5-year height growth had not been measured by using equation [A5] and the relationship

$$H_1 = H_2 \left(\frac{PH2_1}{PH2_2} \right), \quad [A6]$$

where

PH2₁ = predicted height at the start of the growth period, from equation [A5]

$$= 4.5 + \text{EXP} [B_{0,4} + B_{1,4} (\ln DBH_1)], \text{ and}$$

PH2₂ = predicted height at the end of the growth period, from equation [A5]

$$= 4.5 + \text{EXP} [B_{0,4} + B_{1,4} (\ln DBH_2)].$$

If fewer than six trees in a species had H₂ ≥ 20 feet, the height distinction was eliminated. If the combined data for a species provided at least six measured 5-year height growths, equation [A3] was fit to the data and equation [A4] was used to estimate H₁ for trees without a measured 5-year height-growth rate. If the combined data for a species did not have at least six measurements of 5-year height growth but did have at least six trees, equation [A5] was fit to the data and equation [A6] was used to estimate H₁ for trees without a measured 5-year height-growth rate. Finally, if a species had fewer than six trees, the data for the species were combined with data from similar species and appropriate regression equations were developed.

Height to Crown Base

Height to crown base at the start of the growth period (HCB₁) was computed with the following equation:

$$HCB_1 = H_1 - (H_2 - HCB_2)$$

where

HCB₁ = height to crown base at the start of the growth period,

H_1 = total height at the start of the growth period,
 H_2 = total height at the end of the growth period, and
 HCB_2 = height to crown base at the end of the growth period.

This equation is based on the assumption that crown length (i.e., $H - HCB$) is constant for short growth periods.

Expansion Factor

The expansion factor (number of trees per acre represented by a sampled tree) at the start of the

growth period ($EXPAN_1$) was based on DBH_1 , distance to the center of the tree ($DIST$), and the following rules derived from the sampling design:

1. If $DBH_1 \leq 4.0$ inches and $DIST \leq 7.78$ feet, $EXPAN_1 = 229.18$ trees per acre; otherwise, $EXPAN_1 = 0.0$.
2. If $DBH_1 > 4.0$ inches but ≤ 8.0 inches and $DIST \leq 15.56$ feet, $EXPAN_1 = 57.30$ trees per acre; otherwise, $EXPAN_1 = 0.0$.
3. If $DBH_1 > 8.0$ inches, a critical distance ($CDIST$) is first computed by $CDIST = 1.944544 (DBH_1)$. If $DIST \leq CDIST$, $EXPAN_1$ is computed by $EXPAN_1 = 3666.93 (DBH_1)^{-2}$; otherwise, $EXPAN_1 = 0.0$.

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Models of stand growth and yield must include an estimate of mortality. For individual-tree/distance-independent growth-and-yield models, it is necessary (1) to predict the probability of death of individual trees and (2) to keep stand projections within reasonable biological limits (e.g., the maximum basal area that the stand can achieve). In this study, the probability of death of an individual tree is predicted by a logistic equation with coefficients estimated for tree species growing in the mixed-conifer zone of southwest Oregon. Overall stand projections are controlled by use of size-density trajectories that govern the approach of the stand to the maximum size-density line.

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