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UATIONS FOR SIX CONIFER SPECIES IN SOUTHWEST OREGON

David K. Walters
David W. Hann

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The Authors

David K. Walters was a research assistant and David W. Hann is an associate professor of forest biometry, Department of Forest Management, Oregon State University, Corvallis. Walters is currently a graduate research assistant at Virginia Polytechnic Institute and State University, Blacksburg, Virginia.

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ERRATA (Typographical errors) IN

Walters, David K., and David W. Hann. 1986. Taper equations for six conifer species in southwest Oregon. Forest Research Laboratory, Oregon State University, Corvallis. Research Bulletin 56. 41 p.

(1) In the "Analysis" subsection of "Selecting Final Equations" (p. 6), the second sentence of the third paragraph should read:

Nonlinear regression techniques were used to estimate the coefficients c_0, c_1, c_2, and c_3 for each species, and $\widehat{CB}$ was substituted for CB in Equation [14] to form Equation [14b].

(2) In Appendix C (p. 34), the second line of the equation under "Final Equation Form" should read:

$$+ A_1\left[I_2\left(X + I_1\{[(X - 1.0)/(k_1 - 1.0)][X + k_1 (k_1 - X)/(k_1 - 1.0)] - X\}\right) - (X - 1.0)(X - I_2X)\right]$$

(3) In Appendix C (p. 35), the height equation should read:

$$\begin{aligned}\hat{h}_m &= [-B + (B^2 - 4AC)^{1/2}]/2A \\ &= [-B - (B^2 - 4AC)^{1/2}]/2A\end{aligned}$$

(4) In Appendix C (p. 35), the third equation for C should read:

$$C = -[d_m/\widehat{DIB} + (2k_1 - 1.0 + A_2 k_1^2 + A_1 k_1^2)/(k_1 - 1.0)^2]$$

CONTENTS

ABBREVIATIONS

$\wedge$	This "hat" symbol over a variable indicates that the value of that variable is predicted from an equation, rather than actually measured
α_j	Proportion of height to crown base at the j^{th} join point in the expression $(\alpha_j CB - 4.5)/H$
$\overline{B}$	Mean unweighted bias
SE(B)	Standard error of the bias
bd_i	Diameter inside bark at the i^{th} point of interest below breast height, inches
$bd_i/\hat{d}_{1.0}$	Relative diameter below breast height
bh_i	Distance from the ground to the i^{th} point of interest below breast height, feet
CB	Height to live–crown base, feet
d_i	Diameter inside bark at the i^{th} point of interest above breast height, inches
DIB	Diameter inside bark at breast height, inches
d_i/DIB	Relative diameter above breast height (preliminary equations)
$d_i/\widehat{DIB}$	Relative diameter above breast height (final equations)
DOB	Diameter outside bark at breast height, inches
d_m	Merchantable top diameter inside bark, inches
$d_{1.0}$	Diameter inside bark 1.0 foot above the ground, inches
h_i	Height above breast height to the i^{th} point of interest, feet
H_t	Total tree height, feet
H	Total tree height above breast height (H_t – 4.5), feet
h_i/H	Relative height
h_m	Distance between breast height and merchantable top diameter, feet
h_s	Stump height, feet
I_ℓ	Indicator variables
k_j	Relative height at which individual equation segments are joined (join points), where $j = 1$ (upper join point when equation has three segments, or the only join point when equation has two segments), $j = 2$ (lower join point when equation has three segments)
MSE	Mean square error
V_m	Merchantable volume inside bark above breast height to d_m, cubic feet
V_{bbh}	Volume below breast height to any stump height, cubic feet
X	h_i/H (relative height)
Y^*	d_i/DIB [relative diameter above breast height (preliminary equations)]
Y	$d_i/\widehat{DIB}$ [relative diameter above breast height (final equations)]
Y_{bbh}	$bd_i/\hat{d}_{1.0}$ (relative diameter below breast height)

ABSTRACT

Taper equations predicting upper stem diameters inside bark are presented for Douglas-fir, grand fir, white fir, ponderosa pine, sugar pine, and incense-cedar, the six most common species in the mixed conifer zone of southwest Oregon. Fourteen different equations, including 11 segmented polynomial equations, are examined in a preliminary analysis so that the most appropriate form can be identified. The best choice is further modified and any undesirable equation behavior eliminated. Because height to crown base significantly improves the model, an equation predicting height to live-crown base also is developed for foresters lacking actual crown measurements. The final taper equation may be manipulated to yield estimates of merchantable height and volume to any top diameter as well. A summary chart shows how to apply the final equations, which predict diameter inside bark above breast height to any height, merchantable height, and merchantable volume inside bark above breast height to any top diameter inside bark, as well as other equations which predict diameter and merchantable volume inside bark below breast height to any stump height.

INTRODUCTION

Taper equations predicting upper stem diameters inside bark are extremely useful to foresters, providing estimates of cubic-foot volumes inside bark to any merchantable top diameter (Cao et al. 1980), net lumber volume (Heger 1965), and, given a particular product mix, optimal log size (Brink and Von Gadow 1983). Although volume predictions generated from taper equations are not identical to those derived from volume equations, taper equations can still yield accurate estimates of merchantable volumes (Cao et al. 1980).

In this publication, we develop equations for predicting taper above breast height (4.5 feet) for Douglas-fir [Pseudotsuga menziesii (Mirb.) Franco], grand fir [Abies grandis (D. Don) Lindl.], white fir [Abies concolor (Gord. & Glendl.) Hildebr.], ponderosa pine (Pinus ponderosa Laws.), sugar pine (Pinus lambertiana Dougl.), and incense-cedar (Calocedrus decurrens Torr.), the six most common species in the mixed conifer zone of southwest Oregon (Franklin and Dyrness 1973). The taper equations are limited to the stem above breast height in order that the diameter inside bark at the i^{th} point of interest above breast height (d_i) can be constrained to equal the diameter inside bark at breast height (DIB) when height above breast height (h_i) equals zero. Equations predicting upper stem taper can be manipulated to yield estimates of merchantable height and volume inside bark to any top diameter inside bark as well.

Many foresters also require estimates of taper below breast height. Since lower stem form is often assumed to be a frustum of a neiloid[1] (Husch et al. 1982), diameters below breast height can be estimated with an equation assuming a neiloidic form. Such an equation can be applied if at least two diameter points are known or can be estimated. Fortunately, equations that predict diameter inside bark 1.0 foot above the ground (Walters et al. 1985) and at breast height (Larsen and Hann 1985) are available and were included in this analysis so that equations predicting diameter and merchantable volume inside bark below breast height to any stump height also could be developed.

A detailed description of the data and analytical methods used to derive the taper equations is found in the next three sections. Those readers primarily interested in the results should pass directly to the fourth section, "Applying Final Equations" (page 11). Terms are defined and their abbreviations given at first mention in the text; those used throughout this publication are listed for easy reference in "Abbreviations" (page iii).

[1] Use of a neiloid assumes that the diameter at a point on the tree's stem is proportional to the distance from tree tip to that point raised to the 3/2 power.

DATA DESCRIPTION

The data used in this analysis were collected as part of the Growth and Yield Project conducted by the FIR (Forestry Intensified Research) Program. The study area extends from near the California border (42° 10'N) on the south to Cow Creek (43° 00'N) on the north and from the Cascade crest (122° 15'W) on the east to approximately 15 miles west of Glendale (123° 50'W) (Figure 1). Elevation ranges from 900 to 5100 feet, January mean minimum temperature from

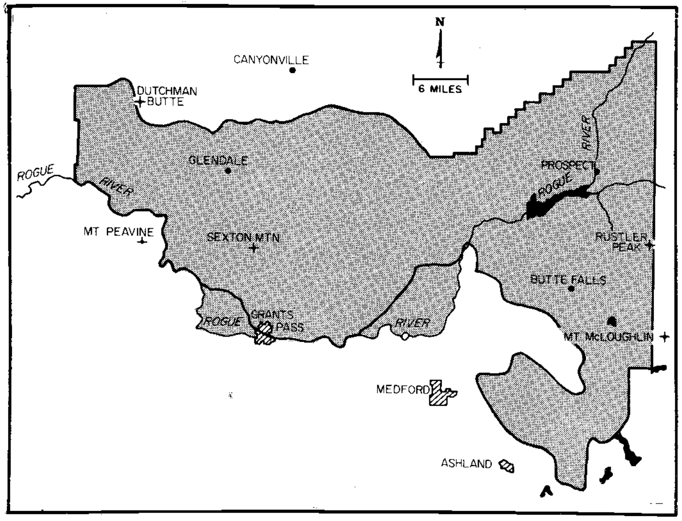

FIGURE 1.
STUDY AREA (SHADED) AND ENVIRONS OF GROWTH AND YIELD PROJECT.

23 to 32°F, and July mean maximum temperature from 79 to 90°F. Annual precipitation varies from 29 to 83 inches, with < 10 percent of the total falling during June, July, and August. Of the 26,441 trees measured, a subsample of 1242 trees representing the dominant, codominant, and intermediate crown classes was selected for analysis. This subsample yielded wide ranges of diameters at breast height outside bark (DOB) and total heights (H_t) (Appendix A).

The trees from the subsample were felled at a stump height of approximately 1.0 foot and sectioned first at breast height and thereafter at approximately 8.4-foot intervals. For each section, diameter inside bark for the longest and shortest axis was measured and the geometric mean of those two diameters calculated. The geometric mean was used because it yields the correct cross-sectional area for both ellipses and circles (Brickell 1976). Total height and height to live-crown base (CB) were also estimated before trees were felled as part of the larger Growth and Yield Project. So that CB might be adjusted to the felled-tree measurements, the estimated standing-tree CB was multiplied by the ratio of felled-tree H_t to standing-tree H_t.

TESTING PRELIMINARY EQUATIONS

Analysis

The diameter and height values for 7255 sections from 682 felled Douglas-fir trees were used to screen preliminary equations to identify the most appropriate equation form. All equations examined are conditioned such that:

$d_i/DIB = 0.0$ when $h_i/H = 1.0$
$d_i/DIB = 1.0$ when $h_i/H = 0.0$

where:
d_i/DIB = relative diameter
h_i/H = relative height, where H = total height above breast height ($H_t - 4.5$ ft).

The four initial forms examined—Equations [1], [2], [3], and [4]—are, respectively, modified versions of equations developed by Biging (1984), Amidon (1984), Bennett and Swindel (1972), and Max and Burkhart (1976). The modifications ensure that the above two conditions are satisfied.

where:

$Y^* = d_i/DIB$ (relative diameter)

$X = h_i/H$ (relative height)

$Y_1 = 1.0 + A_1 X + A_2 X^2$

$Y_2 = B_0 + B_1 X + B_2 X^2$

$Y_3 = C_1 (X - 1.0) + C_2 (X^2 - 1.0)$

$I_1 = I_2 = 0.0$ when $0.0 \leq X \leq k_2$

$I_1 = 1.0, I_2 = 0.0$ when $k_2 < X < k_1$

$I_1 = 0.0, I_2 = 1.0$ when $k_1 \leq X \leq 1.0$

k_j = relative height at which individual equation segments are joined, where j = 1 (upper join point), j = 2 (lower join point)

$A_1, A_2, A_3, B_0, B_1, B_2, C_1, C_2$ = coefficients to be estimated.

$$Y^* = 1.0 + A_1 \ln\{1.0 - [1.0 - EXP(-1.0/A_1)]\, X^{A_2}\} \quad [1]$$

$$Y^* = (H - h_i)/H + A_1 (H^2 - h_i^2)\, h_i/(H^2 \cdot DIB) \quad [2]$$

$$Y^* = (H - h_i)/H + A_1 (H - H_i)\, h_i/DIB + A_2 H (H - h_i)\, h_i/DIB + A_3 (H - h_i)\, h_i (H + h_i + 4.5)/DIB \quad [3]$$

$$Y^* = Y_1 + I_1 (Y_2 - Y_1) + I_2 (Y_3 - Y_1) \quad [4]$$

Equation [4] can be further conditioned such that:

$$Y_1 = Y_2 \text{ when } X = k_2$$
$$Y_2 = Y_3 \text{ when } X = k_1$$
$$\partial Y_1/\partial X = \partial Y_2/\partial X \text{ when } X = k_2$$
$$\partial Y_2/\partial X = \partial Y_3/\partial X \text{ when } X = k_1$$

where $\partial Y_i/\partial X$ = the first derivative of Y_i with respect to X. The first two conditions guarantee that two adjoining segments are equal at the join points, whereas the last two conditions guarantee that adjoining segments are continuous at the join points. Solving for these conditions eliminates B_o, B_1, C_1, and C_2 from Equation [4], giving the final form shown in Appendix B.

Equation [1] must be solved through nonlinear least-squares regression techniques. Because of the large number of observations, this equation was fit to mean relative diameter values calculated for 40 relative height classes. Equations [2] and [3] can be linearized by subtracting the intercept term, and the parameters can then be estimated with linear least-squares regression techniques. Equation [4] can be linearized by first subtracting the intercept term and fixing k_1 and k_2 for a given regression and then manually iterating k_1 and k_2 through a series of regression fits. In this manner, k_1 and k_2 were estimated to the nearest 0.01. The form of Equation [4] seemed amenable to the addition of CB; therefore, the following function of CB was used to model k_1 and k_2 for developing Equation [4b]:

$$k_j = (\alpha_j CB - 4.5)/H$$

where α_j = proportion of height to crown base at the j^{th} join point (j = 1 or 2).

The segmented polynomial Equations [4] and [4b] are composed of three quadratic segments (referred to as a three-quadratic equation) with equal first derivatives at both join points to characterize relative taper. However, numerous other possible combinations of equations could also be derived. For example, two quadratic segments instead of three, or a cubic lower segment or a linear upper segment, could be used instead. In addition, the condition of equal first derivatives could be eliminated. On the basis of these and other alternatives, nine additional segmented polynomial equations were developed for preliminary testing (see Appendix B). These equations were fit to actual DIB values from the Douglas-fir felled-tree subsample. The 14 equations were compared on the basis of their weighted mean square errors (MSEs) and the most promising ones checked graphically for lack of fit by plotting class averages of predicted and actual stem profiles for 12 combinations of DOB and H_t.

Results

The initial screening with the Douglas-fir data eliminated many of the equations. Equations [1] and [2] were eliminated because of their relatively high MSEs (Table 1). Although Equation [4] also had a relatively high MSE, it was retained for comparison purposes. Equation [3] had the lowest MSE of all 14 equations and therefore was retained for further analysis. Equations [6], [7], [9], [12], and [13], which are not constrained to have a continuous first derivative at k_1, were eliminated because they did not fit as well as continuous Equations [4b], [5], [8], [10], and [11]. Of the remaining segmented polynomial equations, Equations [10] and [11] had the lowest MSEs. Of those two, [10] was preferred because [11], a much more complex equation, had several insignificant coefficients. Therefore, Equation [10] was also retained for further modification and analysis.

Discussion

Max and Burkhart (1976) found that a three-quadratic equation was best for describing total stem relative taper. Larson (1963) had earlier suggested that tree form can be decomposed into three distinct sections: the stem within the crown, the branch-free stem above the butt swell, and the region of butt swell itself. However, comparing a two-quadratic equation (such as [5]) and a three-quadratic equation (such as [4b]) suggests that a two-quadratic equation is better at describing taper above breast height (Table 1); Demaerschalk and Kozak (1977) also support using two functions. Although a three-quadratic equation may better describe total stem relative taper, forcing the constraint that d_i = DIB at breast height becomes difficult. A two-quadratic equation also is preferable to a quadratic-linear equation (such as [7]) or a cubic-quadratic equation (such as [8]). Equation [7] has a higher MSE than Equation [5], and Equation [8] converges to Equation [5] in both MSE and parameter estimates.

If the condition that the function be continuous is excluded (thereby allowing Equations [6], [7], [9], [12], and [13]), α_1 is consistently $\geq$ 1.0. This suggests that an actual break in stem form may occur at, or slightly above, crown base. Imposing the condition of continuous first derivatives

TABLE 1.

MEAN SQUARE ERROR OF PRELIMINARY TAPER EQUATIONS FIT TO RELATIVE DIAMETERS INSIDE BARK ABOVE BREAST HEIGHT (Y*) FROM THE DOUGLAS-FIR FELLED-TREE SUBSAMPLE.

Equation number	General description	Join points k_1	k_2	Mean square error
[1]	Modified Biging (1984)	NA	NA	0.002409
[2]	Modified Amidon (1984)	NA	NA	0.001946
[3]	Modified Bennett and Swindel (1972)	NA	NA	0.001660
Segmented polynomials (Max and Burkhart 1976)				
[4]	3-quadratic equation with equal derivatives at both the k_1 and k_2 join points	0.85	0.05	0.002524
[4b]	Same as [4] except for join points	(0.60CB – 4.5)/H	(0.55CB – 4.5)/H	0.001769
[5]	2-quadratic equation with equal derivatives at the k_1 join point	(0.50CB – 4.5)/H	NA	0.001757
[6]	2-quadratic equation with unequal derivatives at the k_1 join point	(1.00CB – 4.5)/H	NA	0.001764
[7]	Quadratic-linear equation with unequal derivatives at the k_1 join point	(1.15CB – 4.5)/H	NA	0.001901
[8]	Cubic-quadratic equation with equal derivatives at the k_1 join point	(0.50CB – 4.5)/H	NA	0.001757
[9]	3-quadratic equation with equal derivatives at the k_2 join point and unequal derivatives at the k_1 join point	(1.00CB – 4.5)/H	(0.05CB – 4.5)/H	0.001742
[10]	Equation [5] when the k_1 join point > 0.0; a single quadratic when the k_1 join point $\leq$ 0.0	(0.50CB – 4.5)/H	NA	0.001743
[11]	Equation [4b] when the k_2 join point > 0.0; Equation [10] when the k_2 join point < 0.0	(0.60CB – 4.5)/H	(0.55CB – 4.5)/H	0.001737
[12]	Equation [9] when the k_2 join point > 0.0; Equation [13] when the k_2 join point < 0.0	(1.00CB – 4.5)/H	(0.05CB – 4.5)/H	0.001737
[13]	Equation [6] when the k_1 join point > 0.0; a single quadratic when the k_1 join point $\leq$ 0.0	(1.00CB – 4.5)/H	NA	0.001760

NA = not applicable; for definitions of other abbreviations, see abbreviations list (p. iii).

forces α_1 to approximately 0.50. Traditionally, the continuous function has been preferred because it can be analytically integrated whereas the discontinuous function cannot. Because no significant advantage was indicated for the discontinuous functions in this preliminary analysis, the added benefit of continuity influenced the selection of a final equation.

Finally, the multiple segmented polynomials (Equations [10]–[13]) were derived with the expectation that, when k_j falls below breast height for a particular stem, the stem can be adequately modeled with a simpler equation. For example, Equation [10], which has a lower MSE than Equation [5], is equal to Equation [5] when k_1 is above breast height; otherwise, [10] reduces to a single quadratic. Apparently, the addition of the single quadratic when k_1 is below breast height is a good one. This approach essentially uses different equations for different-sized trees and appears quite promising.

SELECTING FINAL EQUATIONS

Analysis

On the basis of the preliminary analysis, we decided to further explore the use of Equation [10]:

$$Y = Y_3 + I_2(Y_A - Y_3)$$

where:

$$Y_A = Y_1 + I_1(Y_2 - Y_1)$$
$$Y_1 = 1.0 + A_1 X + A_2 X^2$$
$$Y_2 = B_1 (X - 1.0) + B_2 (X^2 - 1.0)$$
$$Y_3 = 1.0 - X + C_2 (X^2 - X)$$
$$I_1 = 0.0 \text{ when } 0.0 \leq X \leq k_1$$
$$= 1.0 \text{ when } k_1 < X \leq 1.0$$
$$I_2 = 0.0 \text{ when } k_1 \leq 0.0$$
$$= 1.0 \text{ when } k_1 > 0.0$$

This equation is conditioned such that

$$Y_1 = Y_2 \text{ when } X = k_1$$
$$\partial Y_1/\partial X = \partial Y_2/\partial X \text{ when } X = k_1$$

and, further, such that

$$Y_A = Y_3 \text{ when } k_1 \leq 0.0$$

Once conditioned and solved for B_1, B_2, and C_2, the equation can be expressed as

$$Y = Z_o + A_1 Z_1 + A_2 Z_2 \qquad [14]$$

where:

$$Z_o = 1.0 - X + I_2\Big(X + I_1\{[(X - 1.0)/(k_1 - 1.0)][1.0 + (k_1 - X)/(k_1 - 1.0)] - 1.0\}\Big) - (X - 1.0)(X - I_2 X)$$
$$Z_1 = I_2\Big(X + I_1\{[(X - 1.0)/(k_1 - 1.0)][X + k_1(k_1 - X)/(k_1 - 1.0)] - X\}\Big) - (X - 1.0)(X - I_2 X)$$
$$Z_2 = I_2\Big(X^2 + I_1\{k_1[(X - 1.0)/(k_1 - 1.0)][2X - k_1 + k_1(k_1 - X)/(k_1 - 1.0)] - X^2\}\Big)$$

Equations [14] and [3] were then fit to the data for each species (grand fir and white fir were combined into one species group) with $Y = d_i/\widehat{DIB}$, where $\widehat{DIB}$ is predicted DIB obtained from the following equation developed by Larsen and Hann (1985):

$$\widehat{DIB} = a_1 \, DOB^{a_2} \qquad [15]$$

Values of regression coefficients a_1 and a_2 are listed for all six species in Table 2. Predicted DIB was used to develop the final equations because actual DIB is seldom measured by those who may apply the taper equations.

TABLE 2.

REGRESSION COEFFICIENTS FOR PREDICTING DIAMETER INSIDE BARK AT BREAST HEIGHT, BY SPECIES,† EQUATION [15].

Species	Regression coefficients a_1	a_2
Douglas-fir	0.903563	0.989388
Grand/white fir	0.904973	1.000000
Ponderosa pine	0.809427	1.016866
Sugar pine	0.859045	1.000000
Incense-cedar	0.837291	1.000000

† From Larsen and Hann (1985).

Since Equation [14] includes only h_i/H and CB as variables, we believed (on the basis of preliminary graphical analysis) that the goodness of fit could be improved by modeling A_1 and A_2 as functions of H and DOB. Therefore, the following equation was fit:

$$Y = Z_o + (b_{10} + b_{11}T_{11} + b_{12}T_{12} + \dots b_{1i}T_{1i})Z_1$$
$$+ (b_{20} + b_{21}T_{21} + b_{22}T_{22} + \dots b_{2i}T_{2i})\, Z_2$$

where T_i = transformations of H and DOB. Values of all coefficients (b_{10}, b_{11}, ... b_{2i}) were then checked with separate t-tests to determine whether they were significantly different ($p = 0.01$) from 0.0.

It was also necessary to develop a taper equation that did not depend on knowing actual height to crown base. The most logical approach was to predict CB and substitute this predicted value ($\hat{CB}$) into the equation. After various forms were examined, the following was selected:

$$\hat{CB} = H_t/\{1.0 + EXP[c_o + c_1 H_t + c_2(H_t/DOB) + c_3(H_t/DOB)^2]\} \qquad [16]$$

Dividing through by H_t transformed the equation to homogenize the variance. Nonlinear regression techniques were used to estimate the coefficients c_o, c_1, c_2, and c_3 for each species, and $\hat{CB}$ was substituted for CB in Equation [14] to form Equation [14b]. Equation [14b] was then fit to the data from each species.

Equations [14] and [14b] were checked for all six species and Equations [3] and [4] for Douglas-fir only over 10 relative height classes on the basis of the mean unweighted bias ($\overline{B}$), standard error of the bias [SE(B)], and range of the bias. These statistics are defined:

$$\overline{B} = 1/n \sum_{i=1}^{n} (d_i - \hat{d}_i)$$

$$SE(B) = \{ \sum_{i=1}^{n} (d_i - \hat{d}_i)^2 - (1/n) [\sum_{i=1}^{n} (d_i - \hat{d}_i)]^2 \}/(n - p - 1)$$

where:

- n = number of observations
- p = number of parameters
- $\hat{d}_i$ = predicted d_i, calculated by multiplying d_i/DIB from [14] and [14b] by $\hat{DIB}$ from [15].

$\overline{B}$ values were then checked with a t-test (Bard 1974) to determine whether they were significantly different ($p = 0.01$) from 0.0.

Finally, the taper equations for each species were examined for local maxima and minima so that any undesirable behavior, such as bulges in the tree diameter function, could be detected. To do this, a FORTRAN program was developed to calculate precisely which combinations of independent variables resulted in maxima or minima and to give some indication of the severity of the problem. The incidence of local maxima or minima was then compared with the range of the independent variables in both the felled-tree subsample and the overall growth and yield sample. If a particular combination of variables resulted in unacceptable equation behavior, the equation was simplified until a combination producing acceptable behavior was found.

Equations [14] and [14b] can also be manipulated with algebra to yield estimates of merchantable height above breast height to any top diameter inside bark and with calculus to yield estimates of merchantable volume above breast height to any top diameter inside bark. For each species, the volumes predicted from these two equations were checked against actual total stem volumes and merchantable cubic-foot volumes above breast height with the following equation form and a weight of $(\hat{V}_m)^{-2}$ to homogenize the variance:

$$V_m = e_1 \hat{V}_m \qquad [17]$$

where:

- V_m = actual merchantable volume above breast height to a top diameter inside bark of 0.0 (tree top), 2.0, 4.0, or 6.0 inches
- $\hat{V}_m$ = predicted merchantable volume above breast height to a top diameter inside bark of 0.0, 2.0, 4.0, or 6.0 inches (from the integrated forms of Equations [14] and [14b] times $\hat{DIB}$).

A t-test was applied to the regression coefficient e_1 to determine whether it was significantly different ($p = 0.01$) from 1.0.

In addition, other equations were developed for estimating diameter and merchantable volume below breast height. The taper equation for estimating diameter inside bark below breast height to any stump height was derived from the equation of a neiloid frustum given in Husch et al. (1982). In this equation, taper below breast height is constrained to equal taper above breast height at breast height. However, the two equations have not been constrained to be continuous at breast height. The equation for predicting merchantable cubic-foot volume inside bark below breast height was derived by integrating the taper equation predicting diameter inside bark below breast height.

Results

The final form of Equations [14] and [14b], which predict relative diameter inside bark above breast height, is

$$\hat{Y} = Z_o + [b_{1o} + b_{11}(H/DOB) + b_{12}(H/DOB)^2]Z_1 + b_{2o}Z_2$$

A detailed summary of these equations is presented in Appendix C; values of α_1 and regression coefficients b_{1o}, b_{11}, b_{12}, and b_{2o} are given for all six species in Table 3. The MSEs and adjusted coefficients of determination for all species also are found in Table 3. The MSE for Equation [3] fit to predicted rather than actual DIBs from the Douglas-fir data set was 0.001833; note that this value is consistently higher than the MSEs for Equations [14] and [14b] fit to Douglas-fir data (Table 3). This result also held for the remaining species.

The values of regression coefficients for predicting CB, Equation [16], are given in Table 4. Table 5 presents the statistics for checking the bias and precision of Equations [14], [14b], [3], and [4] fit to the Douglas-fir data only, Table 6 the same statistics for Equations [14] and [14b] only fit to the data for the remaining five species.

The algebraic solution of Equations [14] and [14b] to estimate h_i to any d_i (Equations [18] and [18b]) and the integral solution of those same equations to estimate V_m to any d_m (Equations [19] and [19b]) are also found in Appendix C. The values and standard errors of the regression coefficients for Equation [17] are given in Table 7. The resulting t-tests indicated that values of all coefficients were not significantly

TABLE 3.

REGRESSION COEFFICIENTS AND OTHER STATISTICS FOR PREDICTING RELATIVE DIAMETER INSIDE BARK ABOVE BREAST HEIGHT ($\hat{Y}$), BY SPECIES, EQUATIONS [14] AND [14b].

Species	No. of trees	No. of sections	Adjusted coefficient of determination	Mean square error	Regression coefficients b_{1o}	b_{11}	b_{12}	b_{2o}	α_1†
				Actual height to crown base (Equation [14])					
Douglas-fir	682	7255	0.917881	0.001677	−1.308050	0.1736500	−0.0093919	0.229846	0.50
Grand/white fir	187	1953	0.834460	0.001714	−1.784690	0.3254520	−0.0194196	0.769799	0.33
Ponderosa pine	140	1497	0.925210	0.002267	−0.800379	0.0218931	0.00	0.192253	0.60
Sugar pine	92	1053	0.973149	0.001519	−1.151370	0.0692089	0.00	0.081120	0.74
Incense-cedar	141	940	0.930031	0.002995	−1.332420	0.1078950	0.00	0.140898	0.71
				Predicted height to crown base (Equation [14b])					
Douglas-fir	682	7255	0.914052	0.001746	−1.332560	0.1682970	−0.0089899	0.371387	0.50
Grand/white fir	187	1953	0.826004	0.001769	−1.855820	0.3468810	−0.0217170	0.978073	0.33
Ponderosa pine	140	1497	0.930521	0.002070	−0.879137	0.0161367	0.00	0.485846	0.60
Sugar pine	92	1053	0.973544	0.001479	−1.159700	0.0619508	0.00	0.183413	0.74
Incense-cedar	141	940	0.925181	0.003096	−1.332360	0.1040340	0.00	0.198113	0.71

† In the expression $k_1 = (\alpha_1 CB - 4.5)/H$, α_1 is the proportion of height to crown base at the k_1 join point.

different from 1.0. Therefore, the manipulated taper equations provide unbiased volume predictions for top diameters inside bark ranging from 0.0 to 6.0 inches.

Equations estimating diameter (Equation [20]) and merchantable volume (Equations [22] and [22b]) inside bark below breast height and related coefficients are given in Appendix C.

TABLE 4.

REGRESSION COEFFICIENTS AND OTHER STATISTICS FOR PREDICTING HEIGHT TO LIVE-CROWN BASE, BY SPECIES, EQUATION [16].

Species	Number of trees	Adjusted coefficient of determination	Mean square error	Regression coefficients			
				c_0	c_1	c_2	c_3
Douglas-fir	682	0.395474	0.0191054	3.764343	−0.012033	−0.529574	0.017875
Grand/white fir	187	0.421417	0.0239920	3.727414	−0.014599	−0.340757	0.00
Ponderosa pine	140	0.498829	0.0085918	1.795295	−0.007186	−0.229465	0.00
Sugar pine	92	0.497851	0.0132860	2.950704	−0.012390	−0.355704	0.00
Incense-cedar	141	0.306510	0.0226060	3.429804	−0.012321	−0.706241	0.036604

TABLE 5.

STATISTICS FOR CHECKING THE BIAS AND PRECISION OF FINAL TAPER EQUATIONS [14] AND [14b] AND PRELIMINARY EQUATIONS [3] AND [4] ACROSS 10 RELATIVE HEIGHT CLASSES FOR DOUGLAS-FIR.

Relative height class	No. of sections	Mean bias				Standard error of bias				Range of bias			
		[14]	[14b]	[3]	[4]	[14]	[14b]	[3]	[4]	[14]	[14b]	[3]	[4]
0.05	991	−0.1	−0.1	0.0	−0.1	0.6	0.6	0.6	0.6	−6.5 to 1.6	−6.5 to 1.6	−6.5 to 1.7	−6.0 to 1.6
0.15	589	−0.1	−0.1	−0.1	−0.2	0.9	0.9	0.9	0.8	−7.2 to 1.9	−7.1 to 1.9	−7.8 to 1.8	−6.4 to 1.6
0.25	653	0.0	0.0	−0.1	0.0	0.8	0.8	0.9	0.7	−6.3 to 2.1	−6.5 to 2.1	−7.7 to 2.1	−5.4 to 2.4
0.35	600	0.0	0.0	−0.1	0.1	0.8	0.9	1.0	0.8	−5.8 to 2.2	−6.7 to 2.4	−7.9 to 2.1	−5.1 to 2.2
0.45	635	0.1	0.1	0.0	0.2	0.8	0.9	1.0	0.7	−3.9 to 2.3	−5.4 to 2.4	−6.4 to 2.2	−3.9 to 2.1
0.55	640	0.1	0.0	0.0	0.1	0.7	0.8	1.0	0.8	−3.0 to 2.1	−4.6 to 2.6	−5.6 to 2.6	−3.3 to 2.7
0.65	656	0.1	0.0	0.0	0.0	0.7	0.8	1.0	0.8	−2.2 to 2.8	−3.7 to 3.1	−4.1 to 3.2	−3.3 to 3.2
0.75	641	0.0	−0.1	0.0	−0.2	0.7	0.7	1.0	0.8	−2.8 to 2.8	−4.1 to 3.5	−3.8 to 3.1	−4.0 to 2.9
0.85	674	−0.2	−0.2	−0.1	−0.3	0.5	0.6	0.7	0.6	−2.8 to 2.4	−3.9 to 2.1	−3.0 to 2.6	−3.4 to 1.7
0.95	1176	−0.1	−0.1	0.0	−0.1	0.3	0.3	0.3	0.3	−1.9 to 2.2	−1.7 to 2.3	−2.0 to 2.5	−2.0 to 2.3
Overall	7255	0.0	0.0	−0.1	−0.1	0.7	0.7	0.8	0.7	−7.2 to 2.8	−7.1 to 3.5	−7.9 to 3.2	−6.4 to 3.2

TABLE 6.

STATISTICS FOR CHECKING THE BIAS AND PRECISION OF FINAL TAPER EQUATIONS [14] AND [14b] ACROSS 10 RELATIVE HEIGHT CLASSES FOR THE REMAINING FIVE SPECIES.

Relative height class	No. of sections	Mean bias [14]	Mean bias [14b]	Standard error of bias [14]	Standard error of bias [14b]	Range of bias [14]	Range of bias [14b]
				Grand/white fir			
0.05	259	–0.1	–0.1	0.6	0.6	–3.9 to 1.7	–3.9 to 1.6
0.15	171	0.0	0.0	0.7	0.7	–3.0 to 2.1	–3.0 to 2.0
0.25	176	0.1	0.0	0.8	0.8	–2.5 to 3.2	–2.6 to 3.0
0.35	162	0.1	0.1	0.8	0.8	–2.5 to 3.8	–2.6 to 3.6
0.45	170	0.1	0.0	0.8	0.8	–2.8 to 3.1	–2.9 to 2.8
0.55	178	0.1	0.0	0.7	0.7	–1.8 to 3.0	–1.9 to 2.0
0.65	173	0.0	0.0	0.7	0.7	–1.9 to 2.6	–2.2 to 1.6
0.75	184	0.0	–0.1	0.6	0.7	–2.0 to 2.4	–2.0 to 2.4
0.85	173	–0.1	–0.1	0.5	0.5	–1.5 to 1.8	–1.5 to 1.8
0.95	307	0.1	0.1	0.2	0.2	–0.8 to 1.3	–1.2 to 1.3
Overall	1953	0.0	0.0	0.6	0.6	–3.9 to 3.8	–3.9 to 3.6
				Ponderosa pine			
0.05	200	–0.1	–0.1	0.7	0.7	–2.3 to 2.1	–2.2 to 2.1
0.15	118	–0.2	0.0	0.8	0.9	–3.2 to 2.1	–3.1 to 2.5
0.25	141	0.0	0.1	0.8	0.8	–2.7 to 1.9	–2.5 to 2.2
0.35	116	0.0	0.0	0.8	0.8	–2.2 to 1.5	–2.1 to 1.6
0.45	139	0.0	0.0	0.8	0.7	–2.3 to 1.9	–2.3 to 1.5
0.55	131	0.1	0.0	0.8	0.7	–2.0 to 1.5	–2.5 to 1.3
0.65	131	0.1	0.0	0.7	0.7	–1.8 to 1.9	–2.0 to 1.7
0.75	122	0.1	0.0	0.6	0.6	–1.9 to 1.4	–2.0 to 1.3
0.85	162	0.0	0.0	0.8	0.9	–4.7 to 2.0	–5.3 to 1.8
0.95	237	0.1	0.1	0.2	0.2	–0.9 to 1.4	–1.0 to 1.3
Overall	1497	0.0	0.0	0.7	0.7	–4.7 to 2.1	–5.3 to 2.5
				Sugar pine			
0.05	140	–0.4	–0.4	0.8	0.8	–2.9 to 0.8	–2.8 to 0.8
0.15	82	–0.8	–0.7	0.9	0.9	–3.2 to 0.7	–3.1 to 0.8
0.25	96	–0.3	–0.3	0.8	0.8	–2.8 to 0.9	–2.7 to 1.0
0.35	87	–0.1	0.0	0.7	0.7	–2.2 to 1.9	–2.2 to 1.7
0.45	95	0.1	0.1	0.7	0.7	–1.8 to 2.0	–1.9 to 1.9
0.55	89	0.2	0.2	0.7	0.7	–1.2 to 3.5	–1.3 to 3.1
0.65	95	0.2	0.1	0.7	0.7	–1.4 to 2.9	–1.5 to 2.6
0.75	93	0.1	0.0	0.7	0.7	–2.0 to 2.6	–1.8 to 2.5
0.85	107	–0.1	–0.2	0.5	0.5	–1.6 to 1.8	–1.5 to 2.0
0.95	169	0.0	0.0	0.2	0.2	–0.7 to 1.5	–0.9 to 1.5
Overall	1053	–0.1	–0.1	0.7	0.7	–3.2 to 3.5	–3.1 to 3.1

(continued)

(Table 6 cont.)

				Incense-cedar			
0.05	148	−0.1	−0.1	0.6	0.6	−2.5 to 1.5	−2.5 to 1.5
0.15	59	−0.7	−0.6	0.9	0.9	−2.7 to 1.7	−2.7 to 1.6
0.25	68	−0.3	−0.3	0.7	0.7	−2.1 to 0.8	−2.1 to 0.8
0.35	69	−0.1	−0.1	0.7	0.7	−1.8 to 1.9	−1.8 to 1.9
0.45	64	0.0	0.0	0.5	0.6	−1.4 to 1.4	−1.2 to 1.4
0.55	77	0.1	0.1	0.6	0.6	−1.7 to 1.4	−1.7 to 1.3
0.65	68	0.0	0.0	0.5	0.5	−1.3 to 1.4	−1.2 to 1.5
0.75	89	0.1	0.1	0.5	0.5	−1.2 to 1.9	−1.3 to 2.1
0.85	94	−0.1	−0.1	0.3	0.4	−1.2 to 1.1	−1.0 to 1.2
0.95	204	0.0	0.0	0.1	0.1	−0.6 to 0.6	−0.7 to 0.6
Overall	940	−0.1	−0.1	0.5	0.5	−2.7 to 1.9	−2.7 to 2.1

TABLE 7.

REGRESSION COEFFICIENT e_1 (± STANDARD ERROR) OF EQUATION [17] FOR CHECKING THE BIAS IN PREDICTING MERCHANTABLE VOLUME ABOVE BREAST HEIGHT TO VARIOUS TOP DIAMETERS INSIDE BARK WITH THE TAPER EQUATION, BY SPECIES.

Species	Top diameter, inches			
	0.0 (Tree top)	2.0	4.0	6.0
		Actual height to crown base		
Douglas-fir	1.0049 (±0.0957)	1.0063 (±0.1005)	1.0006 (±0.1144)	1.0021 (±0.1435)
Grand/white fir	1.0053 (±0.0965)	1.0043 (±0.1000)	1.0074 (±0.1136)	1.0158 (±0.1371)
Ponderosa pine	0.9892 (±0.1187)	0.9905 (±0.1240)	0.9715 (±0.1430)	0.9880 (±0.1812)
Sugar pine	0.9992 (±0.1114)	1.0031 (±0.1236)	0.9970 (±0.1538)	0.9846 (±0.1157)
Incense-cedar	1.0375 (±0.1267)	1.0525 (±0.1466)	1.0891 (±0.2331)	1.0647 (±0.5283)
		Predicted height to crown base		
Douglas-fir	1.0022 (±0.0994)	1.0034 (±0.1037)	0.9964 (±0.1186)	0.9987 (±0.1487)
Grand/white fir	0.9874 (±0.1041)	0.9865 (±0.1076)	0.9812 (±0.1155)	0.9881 (±0.1520)
Ponderosa pine	1.0135 (±0.1216)	1.0163 (±0.1275)	1.0007 (±0.1439)	1.0164 (±0.1911)
Sugar pine	1.0067 (±0.1086)	1.0106 (±0.1199)	1.0057 (±0.1554)	0.9955 (±0.1156)
Incense-cedar	1.0279 (±0.1261)	1.0418 (±0.1467)	1.0764 (±0.2310)	1.0575 (±0.5422)

Discussion

The variables H/DOB and $(H/DOB)^2$ were selected as modifiers on A_1 in Equations [14] and [14b] because these terms appeared most frequently among the best combinations of modifiers for each species. However, several different situations caused b_{12}, associated with $(H/DOB)^2$, to be set to 0.0. For sugar pine, b_{12} was not significantly different from 0.0. For ponderosa pine and incense-cedar, the equation that included $(H/DOB)^2$ behaved poorly. When the relative taper equations are modified by these terms, they do not always predict that d_i/DIB will monotonically decrease as h_i/H increases. Certain combinations of H/DOB and CB can result in local maxima or minima in the equations. Because the equations should cover the entire range of possible H/DOB and CB values without having local maxima or minima, all possible combinations of these variables were carefully examined for potential problems. The equations for ponderosa pine and incense-cedar yielded illogical taper predictions at H/DOB values within the range of the overall growth and yield sample; therefore, b_{12} was set to 0.0 for these two species. This action appeared to ameliorate the problem.

The resulting final taper equations, [14] and [14b], generally predict monotonically decreasing diameters as h_i increases, within the range of the felled-tree subsample and the larger growth and yield sample. Exceptions occur in grand/white fir and incense-cedar. The grand/white fir equations predict essentially constant midstem diameters for H/DOB values between 8.0 and 9.0 for Equation [14] and between 8.0 and 10.0 for Equation [14b], but only when H exceeds 155 feet. The same problem exists for incense-cedar when H/DOB $\geq$ 11.0 for both Equations [14] and [14b]. The tables in Appendix D present the distribution of H/DOB and crown ratio statistics for the felled-tree subsample and larger growth and yield sample over which Equations [14] and [14b] can be applied.

For Equations [14] and [14b], α_1 was fixed to the same value because the difference between the values for the two equations was inconsequential. Although Equation [14b] has a smaller MSE than Equation [14] for ponderosa and sugar pines, Equation [14] is recommended because it can be extrapolated with greater confidence.

An examination of Tables 5 and 6 reveals that the overall $\bar{B}$ is either 0.0 (Douglas-fir, grand/white fir, ponderosa pine) or 0.1 (sugar pine, incense-cedar) inches for Equations [14] and [14b]. These values, which are not significantly different from 0.0 ($p = 0.01$), compare very favorably with those reported by Biging (1984). Values of $\bar{B}$ and SE(B) both indicate that Equations [14] and [14b] are better choices than either Equation [3] or [4].

A systematic pattern of bias is present in all equations across the range of relative height classes. The equations overestimate d_i near breast height and, with the exception of ponderosa pine, near the top of the stem, and underestimate d_i in the midportion of the stem. Similar patterns of bias were reported by Biging (1984). However, the t-tests indicate that all bias values were not significantly different from 0.0 ($p = 0.01$).

APPLYING FINAL EQUATIONS

This section instructs users in applying the final equations. Table 8 lists the equations by number and brief description, the independent variables used in each equation, and the location within this publication of each equation and its regression coefficients. The accompanying chart provides a convenient summary of instructions according to whether height to live-crown base is known or must be predicted.

TABLE 8.

SUMMARY OF FINAL EQUATIONS, INDEPENDENT VARIABLES USED IN EACH, AND LOCATION OF EQUATIONS AND TABLES OF REGRESSION COEFFICIENTS.

Equation number and name		Independent variables used in equation	Location, by page no. Equation	Coefficients
		Above breast height		
[14]	Predicted relative taper ($\hat{Y}$)	CB, H, DOB, h_i	34	7
[14b]	Predicted relative taper ($\hat{Y}$)	$\widehat{CB}$, H, DOB, h_i	34	7
[15]	Predicted DIB ($\widehat{DIB}$)	DOB	5	5
[16]	Predicted CB ($\widehat{CB}$)	H_t, DOB	6	8
[18]	Predicted merchantable height ($\hat{h}_m$)	CB, H, $\widehat{DIB}$, DOB, d_m	35	7
[18b]	Predicted merchantable height ($\hat{h}_m$)	$\widehat{CB}$, H, $\widehat{DIB}$, DOB, d_m	35	7
[19]	Predicted merchantable cubic-foot volume ($\hat{V}_m$)	CB, H, $\widehat{DIB}$, DOB, $\hat{h}_m$	36	7
[19b]	Predicted merchantable cubic-foot volume ($\hat{V}_m$)	$\widehat{CB}$, H, $\widehat{DIB}$, DOB, $\hat{h}_m$	36	7
		Below breast height		
[20]	Predicted relative taper ($\hat{Y}_{bbh}$)	$\widehat{DIB}$, $\hat{d}_{1.0}$	37	37
[21]	Predicted $d_{1.0}$ ($\hat{d}_{1.0}$)	CB, DOB, H_t	37	37
[21b]	Predicted $d_{1.0}$ ($\hat{d}_{1.0}$)	DOB	37	37
[22]	Predicted cubic-foot volume ($\hat{V}_{bbh}$)	$\widehat{DIB}$, $\hat{d}_{1.0}$ (from Eq. [21]), h_s	38	37
[22b]	Predicted cubic-foot volume ($\hat{V}_{bbh}$)	$\widehat{DIB}$, $\hat{d}_{1.0}$ (from Eq. [21b]), h_s	38	37

For definitions of abbreviations, see abbreviations list (p. iii).

To predict:	Proceed as follows:	
	Height to crown base known	Height to crown base predicted
Above breast height		
Diameter inside bark	Multiply predictions from Equation [14] by predictions from Equation [15].	Multiply predictions from Equation [14b] by predictions from Equation [15]. Equation [14b] uses a prediction from Equation [16] as an independent variable.
Merchantable height to any top diameter inside bark	Predicted from Equation [18], which uses a prediction from Equation [15] as an independent variable.	Predicted from Equation [18b], which uses predictions from Equations [15] and [16] as independent variables.
Merchantable cubic-foot volume inside bark to any top diameter inside bark	Predicted from Equation [19], which uses predictions from Equations [15] and [18] as independent variables.	Predicted from Equation [19b], which uses predictions from Equations [15], [16], and [18b] as independent variables.
Below breast height		
Diameter inside bark	Multiply predictions from Equation [20] by predictions from Equation [21]. Equation [20] also uses predictions from Equations [15] and [21] as independent variables.	Multiply predictions from Equation [20] by predictions from Equation [21b]. Equation [20] also uses predictions from Equations [15] and [21b] as independent variables.
Merchantable cubic-foot volume inside bark to any stump height	Predicted from Equation [22], which uses predictions from Equations [15] and [21] as independent variables.	Predicted from Equation [22b], which uses predictions from Equations [15] and [21b] as independent variables.
Total tree		
Merchantable height above any stump height to any top diameter inside bark	Predicted by adding $(4.5 - h_s)$ to Equation [18].	Predicted by adding $(4.5 - h_s)$ to Equation [18b].
Merchantable cubic-foot volume inside bark to any top diameter inside bark and any stump height	Predicted by adding the prediction from Equation [19] to that from Equation [22].	Predicted by adding the prediction from Equation [19b] to that from Equation [22b].

SUMMARY

In the preliminary analysis, 7255 Douglas-fir sections from 682 trees were used to screen 14 equations predicting relative diameter inside bark above breast height, including 11 segmented polynomials with various specifications and constraints. Ten of these 11 had join points defined as functions of CB. Apparently, adding CB significantly improves the predictive ability of the equations. Equation [10], a two-quadratic equation which reduces to a single quadratic when the join point falls below breast height, appeared to be the best choice, although of all 14 equations, Equation [3] had the lowest MSE.

In the final analysis, Equation [10] was further modified by constraining the two-quadratic equation to equal the single quadratic when the join point was 0.0 (Equation [14]). Further improvement was gained by modeling the parameters of Equation [14] as functions of H/DOB and $(H/DOB)^2$. Unfortunately, modeling the parameters in this fashion, while significantly reducing the MSE, caused undesirable equation behavior in certain cases. Therefore, the equations were carefully examined for local maxima or minima and, when necessary, were simplified. The final form of Equation [14] was then fit to data for the six most common southwest Oregon species in the mixed conifer zone and found to have a lower MSE than Equation [3]. Equations for predicting CB were also developed for foresters who may not include crown measurements in their inventory data. Predicted CB was substituted into Equation [14] to form Equation [14b], which also had a lower MSE than Equation [3]. Equations [14] and [14b] both have an overall $\bar{B}$ which is not significantly different from 0.0. A pattern is apparent in the distribution of $\bar{B}$ across relative height classes; this pattern also occurred in Equations [3] and [4] for Douglas-fir, and similar trends have been reported in several taper equations for second-growth mixed conifer stands (Biging 1984). However, in no case is $\bar{B}$ significantly different from 0.0.

Equations [14] and [14b] were algebraically solved to produce equations for predicting merchantable height above breast height to any top diameter inside bark and also were integrated to produce equations for predicting merchantable volume above breast height to any top diameter inside bark from 0.0 to 6.0 inches without bias.

On the assumption that stem form below breast height is best characterized by a neiloid, other equations were mathematically derived to predict diameter and merchantable volume inside bark below breast height to any stump height. The equations predicting diameter below breast height were constrained to be equal to but not continuous with the equations predicting diameter above breast height at breast height.

LITERATURE CITED

AMIDON, E.L. 1984. A general taper functional form to predict bole volume for five mixed-conifer species in California. Forest Science 30:166–171.

BARD, Y. 1974. Nonlinear parameter estimation. Academic Press, New York. 341 p.

BENNETT, F.A., and B.F. SWINDEL. 1972. Taper curves for planted slash pine. USDA Forest Service, Southeastern Forest Experiment Station, Asheville, North Carolina. Research Note SE-179. 4 p.

BIGING, G.S. 1984. Taper equations for second-growth mixed conifers of northern California. Forest Science 30:1103–1117.

BRICKELL, J.E. 1976. Bias and precision of the Barr and Stroud dendrometer under field conditions. USDA Forest Service, Intermountain Forest and Range Experiment Station, Ogden, Utah. Research Paper INT-186. 46 p.

BRINK, C., and K. VON GADOW. 1983. Modelling stem profiles. South African Forestry Journal 127:23–25.

CAO, Q.V., H.E. BURKHART, and T.A. MAX. 1980. Evaluation of two methods for cubic-volume prediction of loblolly pine to any merchantable limit. Forest Science 26:71–80.

DEMAERSCHALK, J.P., and A. KOZAK. 1977. The whole-bole system: a conditioned dual equation system for precise prediction of tree profiles. Canadian Journal of Forest Research 7:488–497.

FRANKLIN, J.F., and C.T. DYRNESS. 1973. Natural vegetation of Oregon and Washington. USDA Forest Service, Pacific Northwest Forest and Range Experiment Station, Portland, Oregon. General Technical Report PNW-8. 417 p.

HEGER, L. 1965. A trial of Hohenadl's method of stem form and stem volume estimation. Forestry Chronicle: 466–475.

HUSCH, B., C. MILLER, and T.W. BEERS. 1982. Forest mensuration, third edition. John Wiley and Sons, New York. 402 p.

LARSEN, D.R., and D.W. HANN. 1985. Equations for predicting diameter and squared diameter inside bark at breast height for six major conifers of southwest Oregon. Forest Research Laboratory, Oregon State University, Corvallis. Research Note 77. 4 p.

LARSON, P.R. 1963. Stem form development of forest trees. Forest Science Monograph 5. 41 p.

MAX, T.A., and H.E. BURKHART. 1976. Segmented polynomial regression applied to taper equations. Forest Science 22:283–289.

WALTERS, D.K., D.W. HANN, and M.A. CLYDE. 1985. Equations and tables predicting gross total stem volumes in cubic feet for six major conifers of southwest Oregon. Forest Research Laboratory, Oregon State University, Corvallis. Research Bulletin 50. 37 p.

British/metric conversion table

1 foot = 0.3048 meters
1 inch = 2.54 centimeters
1 cubic foot = 0.0283 cubic meters
1 mile = 1,609.3 meters
1°F = 4.5 (°C) + 32.0

APPENDIX A — Felled-Tree Subsample: Distribution by Species across Diameter (DOB) and Height Classes

TABLE A-1.

DISTRIBUTION ACROSS DIAMETER AND HEIGHT CLASSES OF THE DOUGLAS-FIR FELLED-TREE SUBSAMPLE.

DOB	Total Height (ft.)																				
(in.)	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	Total
1	4	1																			5
2	1	7	2																		10
3		3	11	4																	18
4		3	12	12	4																31
5			9	29	5	3	1														47
6			3	19	21	6	4	1													54
7				11	14	5	2	3													35
8				4	9	8	3		1												25
9				5	7	4	7	5		1											29
10					4	8	4	3	4	1	1	1									26
11					2	7	6	11	2	1											29
12					2	3	10	11	3	2	1	1									33
13					1	3	9	7	2	5	3										30
14							1	6	21	3	2										33
15							3	4	6	5	5	1									24
16							2	4	8	8	7										29
17								1	2	8	4	3									18
18							1	1	5	12	5	3	4								31
19									1	3	3	1	1	1							10
20										4	9	3	4	2							22
21								2	2	6	6	1	4								21
22									1	5	5	7	2	1		2					23
23									1	3	2	7	5								18
24											1	3	1	1							6
25										3	3	1	2	2	2						13
26									1		1	2	4	3		1	2				14
27											4	2	1	1							8
28											1		2	4		1					8
29												1	3	3	1						8
30												1			1	1					3
31													1	1							2
32													1		2	1			1		5
33														1		2					3
34														1			1			1	3
35																					
36																					
37													1		1				2		4
38																					
39																					
40																					
41																					
42															1					1	2
43																		1	1		2
44																					
45																					
Total	5	14	37	84	69	47	53	59	60	70	63	38	36	21	8	8	3	1	4	2	682

TABLE A-2.

DISTRIBUTION ACROSS DIAMETER AND HEIGHT CLASSES OF THE GRAND FIR FELLED-TREE SUBSAMPLE.

DOB	Total Height (ft.)																				
(in.)	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	Total
1	1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	1
2	--	1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	1
3	--	--	2	1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	3
4	--	--	1	1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	2
5	--	--	3	4	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	7
6	--	--	1	2	1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	4
7	--	--	--	--	1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	1
8	--	--	--	1	--	5	--	1	--	--	--	--	--	--	--	--	--	--	--	--	7
9	--	--	--	1	4	2	1	--	--	--	--	--	--	--	--	--	--	--	--	--	8
10	--	--	--	--	1	2	--	--	--	--	--	--	--	--	--	--	--	--	--	--	3
11	--	--	--	--	1	1	1	1	--	--	--	--	--	--	--	--	--	--	--	--	4
12	--	--	--	--	--	2	2	1	2	1	--	--	--	--	--	--	--	--	--	--	8
13	--	--	--	--	--	--	1	1	2	--	--	--	--	--	--	--	--	--	--	--	4
14	--	--	--	--	--	1	--	1	--	3	1	--	--	--	--	--	--	--	--	--	6
15	--	--	--	--	--	--	1	1	1	2	3	--	--	--	--	--	--	--	--	--	8
16	--	--	--	--	--	--	2	--	2	2	--	1	1	--	--	--	--	--	--	--	8
17	--	--	--	--	--	--	--	--	1	--	1	1	--	--	--	--	--	--	--	--	3
18	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
19	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
20	--	--	--	--	--	--	--	--	--	--	--	--	1	1	--	--	--	--	--	--	2
21	--	--	--	--	--	--	--	--	--	--	--	--	2	--	--	--	--	--	--	--	2
22	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
23	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
24	--	--	--	--	--	--	--	--	1	--	--	--	2	--	--	--	--	--	--	--	3
25	--	--	--	--	--	--	--	--	--	--	--	1	2	1	--	--	--	--	--	--	4
26	--	--	--	--	--	--	--	--	--	--	--	--	--	1	1	--	--	--	--	--	2
27	--	--	--	--	--	--	--	--	--	--	--	--	--	1	--	--	--	--	--	--	1
28	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	1	--	--	--	--	1
29	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
30	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
31	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
32	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
33	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
34	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
35	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
36	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
37	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
38	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
39	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
40	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
41	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
42	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
43	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	1	--	--	--	--	1
44	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
45	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
Total	1	1	7	10	8	13	8	6	9	8	5	3	8	4	1	2					94

TABLE A-3.

DISTRIBUTION ACROSS DIAMETER AND HEIGHT CLASSES OF THE WHITE FIR FELLED-TREE SUBSAMPLE.

DOB	Total Height (ft.)																				
(in.)	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	Total
1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
2	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
3	--	1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	1
4	--	--	5	1	1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	7
5	--	--	2	4	1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	7
6	--	--	2	1	4	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	7
7	--	--	--	--	2	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	2
8	--	--	--	--	--	2	1	--	1	--	--	--	--	--	--	--	--	--	--	--	4
9	--	--	--	--	--	2	1	--	--	--	--	--	--	--	--	--	--	--	--	--	3
10	--	--	--	--	1	4	1	1	--	--	--	--	--	--	--	--	--	--	--	--	7
11	--	--	--	--	1	2	1	--	1	--	--	--	--	--	--	--	--	--	--	--	5
12	--	--	--	--	--	2	2	--	1	--	--	--	--	--	--	--	--	--	--	--	5
13	--	--	--	--	--	--	--	1	2	1	--	--	--	--	--	--	--	--	--	--	4
14	--	--	--	--	--	--	--	--	2	1	--	1	--	--	--	--	--	--	--	--	4
15	--	--	--	--	--	--	2	1	--	2	--	--	--	--	--	--	--	--	--	--	5
16	--	--	--	--	--	--	--	--	1	--	--	1	--	--	--	--	--	--	--	--	2
17	--	--	--	--	--	--	--	--	1	--	--	--	1	--	--	--	--	--	--	--	2
18	--	--	--	--	--	--	--	--	1	--	--	--	--	--	--	--	--	--	--	--	1
19	--	--	--	--	--	--	--	--	--	--	1	--	--	--	--	--	--	--	--	--	1
20	--	--	--	--	--	--	--	1	1	1	--	1	--	--	--	--	--	--	--	--	4
21	--	--	--	--	--	--	--	--	--	--	--	--	1	--	--	--	--	--	--	--	1
22	--	--	--	--	--	--	--	--	1	--	--	2	--	--	--	--	--	--	--	--	3
23	--	--	--	--	--	--	--	--	--	--	--	--	--	1	1	--	--	--	--	--	2
24	--	--	--	--	--	--	--	--	--	--	--	1	2	1	--	--	--	--	--	--	4
25	--	--	--	--	--	--	--	--	--	--	--	1	--	--	--	--	--	--	--	--	1
26	--	--	--	--	--	--	--	--	--	--	1	1	--	1	--	--	--	--	--	--	3
27	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
28	--	--	--	--	--	--	--	--	--	--	--	--	--	2	--	--	--	--	--	--	2
29	--	--	--	--	--	--	--	--	--	--	--	--	--	--	2	--	--	--	--	--	2
30	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
31	--	--	--	--	--	--	--	--	--	--	--	--	1	--	--	--	--	--	--	--	1
32	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
33	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	1	--	--	--	--	1
34	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
35	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
36	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
37	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
38	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	1	--	--	--	--	1
39	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
40	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
41	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
42	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
43	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
44	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
45	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
Total		1	9	6	10	12	8	4	12	5	2	8	5	5	3	2					92

TABLE A-4.

DISTRIBUTION ACROSS DIAMETER AND HEIGHT CLASSES OF THE PONDEROSA PINE FELLED-TREE SUBSAMPLE.

DOB	Total Height (ft.)																			
(in.)	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	Total
1	--	1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	1
2	--	1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	1
3	--	--	1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	1
4	--	1	--	2	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	3
5	--	--	2	5	1	1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	9
6	--	1	3	4	2	1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	11
7	--	1	--	2	2	1	1	--	--	--	--	--	--	--	--	--	--	--	--	--	7
8	--	--	--	2	1	1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	4
9	--	--	--	1	2	--	--	1	--	--	--	--	--	--	--	--	--	--	--	--	4
10	--	--	--	--	--	2	1	--	2	--	--	--	--	--	--	--	--	--	--	--	5
11	--	--	--	--	2	2	3	4	1	--	--	--	--	--	--	--	--	--	--	--	12
12	--	--	--	--	--	1	1	3	--	--	--	--	--	--	--	--	--	--	--	--	5
13	--	--	--	--	--	--	--	2	1	2	--	--	--	--	--	--	--	--	--	--	5
14	--	--	--	--	1	1	--	2	2	--	--	--	--	--	--	--	--	--	--	--	6
15	--	--	--	--	--	1	3	--	2	1	--	2	--	--	--	--	--	--	--	--	9
16	--	--	--	--	--	--	3	1	1	--	--	--	--	--	--	--	--	--	--	--	5
17	--	--	--	--	--	--	--	1	1	2	2	1	--	--	--	--	--	--	--	--	7
18	--	--	--	--	--	--	--	1	--	2	--	1	--	--	--	--	--	--	--	--	4
19	--	--	--	--	--	--	--	--	1	1	2	--	--	--	--	--	--	--	--	--	4
20	--	--	--	--	--	--	--	--	2	--	2	1	1	--	--	--	--	--	--	--	6
21	--	--	--	--	--	--	--	1	1	--	--	3	--	--	--	--	--	--	--	--	5
22	--	--	--	--	--	--	--	--	1	--	1	3	--	--	--	--	--	--	--	--	5
23	--	--	--	--	--	--	--	--	1	--	1	--	1	1	--	--	--	--	--	--	4
24	--	--	--	--	--	--	--	--	--	--	--	--	2	--	--	--	--	--	--	--	2
25	--	--	--	--	--	--	--	--	--	--	2	1	--	--	--	--	--	--	--	--	3
26	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
27	--	--	--	--	--	--	--	--	--	--	--	--	3	--	1	--	--	--	--	--	4
28	--	--	--	--	--	--	--	--	--	--	--	--	1	--	--	--	--	--	--	--	1
29	--	--	--	--	--	--	--	--	--	--	--	1	--	1	--	--	--	--	--	--	2
30	--	--	--	--	--	--	--	--	--	--	--	--	1	--	--	--	--	--	--	--	1
31	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
32	--	--	--	--	--	--	--	--	--	--	--	--	1	--	--	1	--	--	--	--	2
33	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
34	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
35	--	--	--	--	--	--	--	--	--	--	--	--	1	--	--	--	--	--	--	--	1
36	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	1	--	1
37	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
38	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
39	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
40	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
41	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
42	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
43	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
44	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
45	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
Total		5	6	16	11	11	12	16	16	8	10	13	11	2	1	1			1		140

TABLE A-5.

DISTRIBUTION ACROSS DIAMETER AND HEIGHT CLASSES OF THE SUGAR PINE FELLED-TREE SUBSAMPLE.

DOB	Total Height (ft.)																				
(in.)	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	Total
1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
2	1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	1
3	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
4	--	1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	1
5	--	1	3	1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	5
6	--	--	--	2	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	2
7	--	--	1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	1
8	--	--	--	1	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	1
9	--	--	--	1	1	--	1	--	--	--	--	--	--	--	--	--	--	--	--	--	3
10	--	--	--	1	2	2	1	--	--	--	--	--	--	--	--	--	--	--	--	--	6
11	--	--	--	1	--	1	--	2	--	--	--	--	--	--	--	--	--	--	--	--	4
12	--	--	--	--	1	2	1	1	2	--	--	--	--	--	--	--	--	--	--	--	7
13	--	--	--	--	--	--	--	1	3	--	--	--	--	--	--	--	--	--	--	--	4
14	--	--	--	--	--	--	2	--	1	1	--	--	--	--	--	--	--	--	--	--	4
15	--	--	--	--	--	1	1	1	3	2	--	--	--	--	--	--	--	--	--	--	8
16	--	--	--	--	--	--	--	--	2	--	--	--	--	--	--	--	--	--	--	--	2
17	--	--	--	--	--	--	--	--	--	2	1	--	--	--	--	--	--	--	--	--	3
18	--	--	--	--	--	--	--	1	1	--	--	--	--	--	--	--	--	--	--	--	2
19	--	--	--	--	--	--	--	2	--	1	--	--	--	--	--	--	--	--	--	--	3
20	--	--	--	--	--	--	--	--	--	2	--	--	1	--	--	--	--	--	--	--	3
21	--	--	--	--	--	--	--	--	--	--	--	--	1	--	--	--	--	--	--	--	1
22	--	--	--	--	--	--	--	--	1	1	--	--	--	--	--	--	--	--	--	--	2
23	--	--	--	--	--	--	--	--	1	1	--	1	--	--	--	--	--	--	--	--	3
24	--	--	--	--	--	--	--	--	--	1	2	1	--	--	--	--	--	--	--	--	4
25	--	--	--	--	--	--	--	--	--	--	--	2	--	--	--	--	--	--	--	--	2
26	--	--	--	--	--	--	--	--	--	--	--	--	2	1	--	--	--	--	--	--	3
27	--	--	--	--	--	--	--	--	--	--	2	1	1	1	--	--	--	--	--	--	5
28	--	--	--	--	--	--	--	--	--	--	--	--	--	1	--	--	--	--	--	--	1
29	--	--	--	--	--	--	--	--	--	--	1	--	--	--	1	--	--	--	--	--	2
30	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
31	--	--	--	--	--	--	--	--	--	--	--	1	2	--	--	--	--	--	--	--	3
32	--	--	--	--	--	--	--	--	--	--	1	1	--	--	--	--	--	--	--	--	2
33	--	--	--	--	--	--	--	--	--	--	--	--	1	--	--	--	--	--	--	--	1
34	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
35	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	1	--	--	1
36	--	--	--	--	--	--	--	--	--	--	--	1	--	--	--	--	--	--	--	--	1
37	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
38	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
39	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
40	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
41	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
42	--	--	--	--	--	--	--	--	--	--	--	--	--	--	1	--	--	--	--	--	1
43	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
44	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
45	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	
Total	1	2	4	7	4	6	6	8	14	11	7	8	8	3	2			1			92

TABLE A-6.

DISTRIBUTION ACROSS DIAMETER AND HEIGHT CLASSES OF THE INCENSE-CEDAR FELLED-TREE SUBSAMPLE.

DOB	Total Height (ft.)																				
(in.)	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	Total
1	3																				3
2	2	1																			3
3	2	7																			9
4		8	3	1																	12
5		5	7																		12
6		4	7	1																	12
7		2	7	6	1																16
8			2	4	2		1														9
9			1	2	2																5
10				1	2	1		1													5
11				1	1	3	1														6
12			1		1	1	2														5
13				2		2															4
14					3	1	1														5
15					1		2	1													4
16				1		2	1														4
17					1	3	2		2												8
18					1		1	1		1											4
19							2	1													3
20																					
21							1	1	1												3
22																					
23								1	1												2
24																					
25									1												1
26							1			1											2
27									1												1
28								1	1												2
29																					
30																					
31																					
32																					
33												1									1
34																					
35																					
36																					
37																					
38																					
39																					
40																					
41																					
42																					
43																					
44																					
45																					
Total	7	27	28	19	15	13	15	7	7	2		1									141

APPENDIX B —

Preliminary Segmented Polynomial Taper Equations: Descriptions, Equation Forms, Conditions

EQUATIONS [4] AND [4b]

Description

Three quadratic segments joined at two points. The segments are equal and have equal first derivatives at both join points.

Preliminary Equation Form

$$Y^* = Y_1 + I_1 (Y_2 - Y_1) + I_2 (Y_3 - Y_1)$$

where:

$$Y_1 = 1.0 + A_1 X + A_2 X^2$$
$$Y_2 = B_0 + B_1 X + B_2 X^2$$
$$Y_3 = C_1 (X - 1.0) + C_2 (X^2 - 1.0)$$
$$\begin{aligned} I_1 &= 0.0 \text{ and } I_2 = 0.0 \text{ when } 0.0 \leq X \leq k_2 \\ &= 1.0 \text{ and } I_2 = 0.0 \text{ when } k_2 < X < k_1 \\ &= 0.0 \text{ and } I_2 = 1.0 \text{ when } k_1 \leq X \leq 1.0 \end{aligned}$$

$A_1, A_2, B_0, B_1, B_2, C_1, C_2$ = regression coefficients to be estimated.

For definitions of other abbreviations, see abbreviations list (p. iii), main text.

Conditions:

$$Y_1 = Y_2 \text{ and } \partial Y_1/\partial X = \partial Y_2/\partial X \text{ when } X = k_2$$
$$Y_2 = Y_3 \text{ and } \partial Y_2/\partial X = \partial Y_3/\partial X \text{ when } X = k_1$$

Final Equation Form

$$\begin{aligned} Y^* = {} & 1.0 + I_2 \{[(X - 1.0)/(k_1 - 1.0)] [1.0 + (k_1 - X)/(k_1 - 1.0)] - 1.0\} \\ & + A_1\left(X + I_2 \{[(X - 1.0)/(k_1 - 1.0)] [X + k_1 (k_1 - X)/(k_1 - 1.0)] - X\}\right) \\ & + A_2\left(X^2 - I_1(X - k_2)^2 + I_2\{[(X - 1.0)/(k_1 - 1.0)][k_2(2X - k_2) + (k_1 - X)(2k_1k_2 - k_2^2)/(k_1 - 1.0)] - X^2\}\right) \\ & + B_2\left(I_1(X - k_2)^2 + I_2\{[(X - 1.0)/(k_1 - 1.0)][k_2^2 - k_1^2 - 2X(k_2 - k_1) + (k_1 - X)(k_1 - k_2)^2/(k_1 - 1.0)]\}\right) \end{aligned}$$

where:

k_1 = constant for Equation [4] = $(\alpha_1 CB - 4.5)/H$ for Equation [4b]
k_2 = constant for Equation [4] = $(\alpha_2 CB - 4.5)/H$ for Equation [4b]
α_1 = proportion of height to crown base at the upper join point
α_2 = proportion of height to crown base at the lower join point.

EQUATION [5]

Description

Two quadratic segments joined at one point. The two segments are equal and have equal first derivatives at the join point.

Preliminary Equation Form

$$Y^* = Y_1 + I_1 (Y_2 - Y_1)$$

where:

$Y_1 = 1.0 + A_1 X + A_2 X^2$
$Y_2 = B_1 (X - 1.0) + B_2 (X^2 - 1.0)$
$I_1 = 0.0$ when $0.0 \leq X \leq k_1$
$= 1.0$ when $k_1 < X \leq 1.0$
A_1, A_2, B_1, B_2 = regression coefficients to be estimated.
For definitions of other abbreviations, see abbreviations list (p. iii), main text.

Conditions:

$Y_1 = Y_2$ and $\partial Y_1/\partial X = \partial Y_2/\partial X$ when $X = k_1$

Final Equation Form

$$\begin{aligned} Y^* = {} & 1.0 + I_1 \{[(X - 1.0)/(k_1 - 1.0)][1.0 + (k_1 - X)/(k_1 - 1.0)] - 1.0\} \\ & + A_1\left(X + I_1 \{[(X - 1.0)/(k_1 - 1.0)][X + k_1 (k_1 - X)/(k_1 - 1.0)] - X\}\right) \\ & + A_2\left(X^2 + I_1 \{k_1 [(X - 1.0)/(k_1 - 1.0)][2X - k_1 + k_1 (k_1 - X)/(k_1 - 1.0)] - X^2\}\right) \end{aligned}$$

EQUATION [6]

Description

Two quadratic segments joined at one point. The two segments are equal but have unequal first derivatives at the join point.

Preliminary Equation Form

$$Y^* = Y_1 + I_1 (Y_2 - Y_1)$$

where:

$Y_1 = 1.0 + A_1 X + A_2 X^2$

$Y_2 = B_1 (X - 1.0) + B_2 (X^2 - 1.0)$

$I_1 = 0.0$ when $0.0 \leq X \leq k_1$

$= 1.0$ when $k_1 < X \leq 1.0$

A_1, A_2, B_1, B_2 = regression coefficients to be estimated.

For definitions of other abbreviations, see abbreviations list (p. iii), main text.

Conditions:

$$Y_1 = Y_2 \text{ when } X = k_1$$

Final Equation Form

$$\begin{aligned} Y^* = {} & 1.0 + I_1 \{[(X - 1.0)/(k_1 - 1.0)] - 1.0\} \\ & + A_1\left(X + I_1 \{k_1 [(X - 1.0)/(k_1 - 1.0)] - X\}\right) \\ & + A_2\left(X^2 + I_1 \{k_1^2 (X - 1.0)/(k_1 - 1.0) - X^2\}\right) \\ & + B_2 I_1 [(k_1 - X)(1.0 - X)] \end{aligned}$$

EQUATION [7]

Description

A quadratic lower segment and a linear upper segment joined at one point. The segments are equal but have unequal first derivatives at the join point.

Preliminary Equation Form

$$Y^* = Y_1 + I_1 (Y_2 - Y_1)$$

where:

$Y_1 = 1.0 + A_1 X + A_2 X^2$

$Y_2 = [1.0 + B_1 k_1 + B_2 k_1^2] [(X - 1.0)/(k_1 - 1.0)]$

$I_1 = 0.0$ when $0.0 \leq X \leq k_1$

$= 1.0$ when $k_1 < X \leq 1.0$

A_1, A_2, B_1, B_2 = regression coefficients to be estimated.

For definitions of other abbreviations, see abbreviations list (p. iii), main text.

Conditions:

$Y_1 = Y_2$ when $X = k_1$

Final Equation Form

$$Y^* = 1.0 + I_1 \{[(X - 1.0)/(k_1 - 1.0)] - 1.0\}$$
$$+ A_1 \left(X + I_1 \{k_1 [(X - 1.0)/(k_1 - 1.0)] - X\}\right)$$
$$+ A_2 \left(X^2 + I_1 \{k_1^2 [(X - 1.0)/(k_1 - 1.0)] - X^2\}\right)$$

EQUATION [8]

Description

A cubic lower segment and a quadratic upper segment joined at one point. The segments are equal and have equal first derivatives at the join point.

Preliminary Equation Form

$$Y^* = Y_1 + I_1 (Y_2 - Y_1)$$

where:

$Y_1 = 1.0 + A_1 X + A_2 X^2 + A_3 X^3$

$Y_2 = B_1 (X - 1.0) + B_2 (X^2 - 1.0)$

$I_1 = 0.0$ when $0.0 \leq X \leq k_1$

$= 1.0$ when $k_1 < X \leq 1.0$

A_1, A_2, A_3, B_1, B_2 = regression coefficients to be estimated.

For definitions of other abbreviations, see abbreviations list (p. iii), main text.

Conditions:

$$Y_1 = Y_2 \text{ and } \partial Y_1/\partial X = \partial Y_2/\partial X \text{ when } X = k_1$$

Final Equation Form

$$\begin{aligned}
Y^* = {} & 1.0 + I_1 \{[(X - 1.0)/(k_1 - 1.0)]\,[1.0 + (k_1 - X)/(k_1 - 1.0)] - 1.0\} \\
& + A_1\left(X + I_1 \{[(X - 1.0)/(k_1 - 1.0)]\,[X + k_1 (k_1 - X)/(k_1 - 1.0)] - X\}\right) \\
& + A_2\left(X^2 + I_1 \{k_1 [(X - 1.0)/(k_1 - 1.0)]\,[2X - k_1 + k_1 (k_1 - X)/(k_1 - 1.0)] - X^2\}\right) \\
& + A_3\left(X^3 + I_1 \{k_1^2 [(X - 1.0)/(k_1 - 1.0)]\,[3X - 2k_1 + k_1 (k_1 - X)/(k_1 - 1.0)] - X^3\}\right)
\end{aligned}$$

EQUATION [9]

Description

Three quadratic segments joined at two points. The lower and middle segments are equal and have equal first derivatives at the lower join point. The middle and upper segments are equal but have unequal first derivatives at the upper join point.

Preliminary Equation Form

$$Y^* = Y_1 + I_1 (Y_2 - Y_1) + I_2 (Y_3 - Y_1)$$

where:

$$Y_1 = 1.0 + A_1 X + A_2 X^2$$
$$Y_2 = B_0 + B_1 X + B_2 X^2$$
$$Y_3 = C_1 (X - 1.0) + C_2 (X^2 - 1.0)$$
$$\begin{aligned} I_1 &= 0.0 \text{ and } I_2 = 0.0 \text{ when } 0.0 \leq X \leq k_2 \\ &= 1.0 \text{ and } I_2 = 0.0 \text{ when } k_2 < X < k_1 \\ &= 0.0 \text{ and } I_2 = 1.0 \text{ when } k_1 \leq X \leq 1.0 \end{aligned}$$

$A_1, A_2, B_0, B_1, B_2, C_1, C_2$ = regression coefficients to be estimated.

For definitions of other abbreviations, see abbreviations list (p. iii), main text.

Conditions:

$$Y_1 = Y_2 \text{ and } \partial Y_1/\partial X = \partial Y_2/\partial X \text{ when } X = k_2$$
$$Y_2 = Y_3 \text{ when } X = k_1$$

Final Equation Form

$$\begin{aligned} Y^* = {} & 1.0 + I_2 \{[(X - 1.0)/(k_1 - 1.0)] - 1.0\} \\ & + A_1\left(X + I_2 \{k_1 [(X - 1.0)/(k_1 - 1.0)] - X\}\right) \\ & + A_2 \{X^2 - I_1 (X - k_2)^2 + I_2 [k_2 (X - 1.0)(2k_1 - k_2)/(k_1 - 1.0) - X^2]\} \\ & + B_2 \{I_1 (X - k_2)^2 + I_2 [(X - 1.0)(k_2 - k_1)^2/(k_1 - 1.0)]\} \\ & + C_2 I_2 (X - k_1) (X - 1.0) \end{aligned}$$

EQUATION [10]

Description

This equation is equal to Equation [5] when $k_1 > 0.0$. Otherwise, it reduces to a single quadratic equation.

Preliminary Equation Form

$$Y^* = Y_3 + I_2(Y_A - Y_3)$$

where:

$$Y_A = Y_1 + I_1(Y_2 - Y_1)$$
$$Y_1 = 1.0 + A_1 X + A_2 X^2$$
$$Y_2 = B_1 (X - 1.0) + B_2 (X^2 - 1.0)$$
$$Y_3 = 1.0 - X + C_2 (X^2 - X)$$
$$I_1 = 0.0 \text{ when } 0.0 \leq X \leq k_1$$
$$= 1.0 \text{ when } k_1 < X < 1.0$$
$$I_2 = 0.0 \text{ when } k_1 \leq 0.0$$
$$= 1.0 \text{ when } 0.0 < k_1$$

$A, A_1, A_2, B_1, B_2, C_2$ = regression coefficients to be estimated.

For definitions of other abbreviations, see abbreviations list (p. iii), main text.

Conditions:

$$Y_1 = Y_2 \text{ and } \partial Y_1/\partial X = \partial Y_2/\partial X \text{ when } X = k_1$$

Final Equation Form

$$\begin{aligned}
Y^* = {} & 1.0 - X + I_2\Big(X + I_1 \{[(X - 1.0)/(k_1 - 1.0)] [1.0 + (k_1 - X)/(k_1 - 1.0)] - 1.0\}\Big) \\
& + A_1 I_2\Big(X + I_1 \{[(X - 1.0)/(k_1 - 1.0)] [X + k_1 (k_1 - X)/(k_1 - 1.0)] - X\}\Big) \\
& + A_2 I_2\Big(X_2 + I_1 \{k_1 [(X - 1.0)/(k_1 - 1.0)] [2X - k_1 + k_1 (k_1 - X)/(k_1 - 1.0)] - X^2\}\Big) \\
& + C_2 (X - 1.0) (X - I_2 X)
\end{aligned}$$

EQUATION [11]

Description

This equation is equal to Equation [4b] when $k_2 > 0.0$. When $k_2 < 0.0$ and $k_1 > 0.0$, the equation reduces to Equation [10].

Preliminary Equation Form

$$Y^* = Y_6 + I_4 (Y_A - Y_6) + I_5 (Y_B - Y_6)$$

where:

$$Y_A = Y_1 + I_1 (Y_2 - Y_1) + I_2 (Y_3 - Y_1)$$
$$Y_B = Y_4 + I_3 (Y_5 - Y_4)$$
$$Y_1 = 1.0 + A_1 X + A_2 X^2$$
$$Y_2 = B_0 + B_1 X + B_2 X^2$$
$$Y_3 = C_1 (X - 1.0) + C_2 (X^2 - 1.0)$$
$$Y_4 = 1.0 + D_1 X + D_2 X^2$$
$$Y_5 = E_1 (X - 1.0) + E_2 (X^2 - 1.0)$$
$$Y_6 = 1.0 - X + F_2 (X^2 - X)$$
$$\begin{aligned} I_1 &= 0.0 \text{ and } I_2 = 0.0 \text{ when } 0.0 \leq X \leq k_2 \\ &= 1.0 \text{ and } I_2 = 0.0 \text{ when } k_2 < X < k_1 \\ &= 0.0 \text{ and } I_2 = 1.0 \text{ when } k_1 \leq X \leq 1.0 \end{aligned}$$
$$\begin{aligned} I_3 &= 0.0 \text{ when } 0.0 \leq X \leq k_1 \\ &= 1.0 \text{ when } k_1 < X < 1.0 \end{aligned}$$
$$\begin{aligned} I_4 &= 0.0 \text{ and } I_5 = 0.0 \text{ when } k_1 \leq 0.0 \\ &= 0.0 \text{ and } I_5 = 1.0 \text{ when } k_2 \leq 0.0 < k_1 \\ &= 1.0 \text{ and } I_5 = 0.0 \text{ when } 0.0 < k_2 \end{aligned}$$

$A_1, A_2, B_0, B_1, B_2, C_1, C_2, D_1, D_2, E_1, E_2, F_2$ = regression coefficients to be estimated.
For definitions of other abbreviations, see abbreviations list (p. iii), main text.

Conditions:

$Y_1 = Y_2$ and $\partial Y_1/\partial X = \partial Y_2/\partial X$ when $X = k_2$
$Y_2 = Y_3$ and $\partial Y_2/\partial X = \partial Y_3/\partial X$ when $X = k_1$
$Y_4 = Y_5$ and $\partial Y_4/\partial X = \partial Y_5/\partial X$ when $X = k_1$

Final Equation Form

$$
\begin{aligned}
Y^* = {} & 1.0 - X + I_4\Big(X + I_2\ \{[(X - 1.0)/(k_1 - 1.0)]\,[1.0 + (k_1 - X)/(k_1 - 1.0)] - 1.0\}\Big) \\
& + I_5\Big(X + I_3\ \{[(X - 1.0)/(k_1 - 1.0)]\,[1.0 + (k_1 - X)/(k_1 - 1.0)] - 1.0\}\Big) \\
& + A_1 I_4\Big(X + I_2\ \{[(X - 1.0)/(k_1 - 1.0)]\,[X + k_1 (k_1 - X)/(k_1 - 1.0)] - X\}\Big) \\
& + A_2 I_4\Big(X^2 - I_1 (X - k_2)^2 + I_2 \{[(X - 1.0)/(k_1 - 1.0)][k_2 (2X - k_2) + (k_1 - X)(2k_2 k_1 - k_2^2)/(k_1 - 1.0)] - X^2\}\Big) \\
& + B_2 I_4\Big(I_1 (X - k_2)^2 + I_2 \{[(X - 1.0)/(k_1 - 1.0)][k_2^2 - k_1^2 - 2X(k_2 - k_1) + (k_1 - X)(k_1 - k_2)^2/(k_1 - 1.0)]\}\Big) \\
& + D_1 I_5\Big(X + I_3\ \{[(X - 1.0)/(k_1 - 1.0)]\,[X + k_1 (k_1 - X)/(k_1 - 1.0)] - X\}\Big) \\
& + D_2 I_5\Big(X^2 + I_3\ \{k_1[(X - 1.0)/(k_1 - 1.0)][2X - k_1 - k_1 (X - k_1)/(k_1 - 1.0)] - X^2\}\Big) \\
& + F_2 (X - 1.0)(X - I_4 X - I_5 X)
\end{aligned}
$$

EQUATION [12]

Description

This equation is equal to Equation [9] when $k_2 > 0.0$. When $k_2 < 0.0$ and $k_1 > 0.0$, the equation reduces to Equation [13].

Preliminary Equation Form

$$Y^* = Y_6 + I_4 (Y_A - Y_6) + I_5 (Y_B - Y_6)$$

where:

$$Y_A = Y_1 + I_1 (Y_2 - Y_1) + I_2 (Y_3 - Y_1)$$
$$Y_B = Y_4 + I_3 (Y_5 - Y_4)$$
$$Y_1 = 1.0 + A_1 X + A_2 X^2$$
$$Y_2 = B_0 + B_1 X + B_2 X^2$$
$$Y_3 = C_1 (X - 1.0) + C_2 (X^2 - 1.0)$$
$$Y_4 = 1.0 + D_1 X + D_2 X^2$$
$$Y_5 = E_1 (X - 1.0) + E_2 (X^2 - 1.0)$$
$$Y_6 = 1.0 - X + F_2 (X^2 - X)$$

$$\begin{aligned} I_1 &= 0.0 \text{ and } I_2 = 0.0 \text{ when } 0.0 \le X \le k_2 \\ &= 1.0 \text{ and } I_2 = 0.0 \text{ when } k_2 < X < k_1 \\ &= 0.0 \text{ and } I_2 = 1.0 \text{ when } k_1 \le X \le 1.0 \end{aligned}$$

$$\begin{aligned} I_3 &= 0.0 \text{ when } 0.0 \le X \le k_1 \\ &= 1.0 \text{ when } k_1 < X < 1.0 \end{aligned}$$

$$\begin{aligned} I_4 &= 0.0 \text{ and } I_5 = 0.0 \text{ when } k_1 \le 0.0 \\ &= 0.0 \text{ and } I_5 = 1.0 \text{ when } k_2 \le 0.0 < k_1 \\ &= 1.0 \text{ and } I_5 = 0.0 \text{ when } 0.0 < k_2 \end{aligned}$$

$A_1, A_2, B_0, B_1, B_2, C_1, C_2, D_1, D_2, E_1, E_2, F_2$ = regression coefficients to be estimated.

For definitions of other abbreviations, see abbreviations list (p. iii), main text.

Conditions:

$Y_1 = Y_2$ and $\partial Y_1/\partial X = \partial Y_2/\partial X$ when $X = k_2$

$Y_2 = Y_3$ when $X = k_1$

$Y_4 = Y_5$ when $X = k_1$

Final Equation Form

$$
\begin{aligned}
Y^* = {} & 1.0 - X + I_4\left(X + I_2 \{[(X - 1.0)/(k_1 - 1.0)] - 1.0\}\right) \\
& + I_5\left(X + I_3 \{[(X - 1.0)/(k_1 - 1.0)] - 1.0\}\right) \\
& + A_1 I_4\left(X + I_2 \{k_1 [(X - 1.0)/(k_1 - 1.0)] - X\}\right) \\
& + A_2 I_4 \{X^2 - I_1 (X - k_2)^2 + I_2 [k_1 (X - 1.0) (2k_1 - k_2)/(k_1 - 1.0) - X^2]\} \\
& + B_2 I_4\left(I_1 (X - k_2)^2 + I_2 \{[(X - 1.0)/(k_1 - 1.0)] [(k_2 - k_1)^2]\}\right) \\
& + C_2 I_4 I_2 (X - k_1) (X - 1.0) \\
& + D_1 I_5\left(X + I_3 \{k_1 [(X - 1.0)/(k_1 - 1.0)] - X\}\right) \\
& + D_2 I_5\left(X^2 + I_3 \{k_1^2 [(X - 1.0)/(k_1 - 1.0)] - X^2\}\right) \\
& + E_2 I_5 I_3 (k_1 - X) (1.0 - X) \\
& + F_2 (X - 1.0) (X - I_4 X - I_5 X)
\end{aligned}
$$

EQUATION [13]

Description

This equation is equal to Equation [6] when $k_1 > 0.0$. Otherwise, it reduces to a single quadratic equation.

Preliminary Equation Form

$$Y^* = Y_3 + I_2(Y_A - Y_3)$$

where:

$$Y_A = Y_1 + I_1(Y_2 - Y_1)$$
$$Y_1 = 1.0 + A_1 X + A_2 X^2$$
$$Y_2 = B_1 (X - 1.0) + B_2 (X^2 - 1.0)$$
$$Y_3 = 1.0 - X + C_2 (X^2 - X)$$
$$I_1 = 0.0 \text{ when } 0.0 \leq X \leq k_1$$
$$= 1.0 \text{ when } k_1 < X \leq 1.0$$
$$I_2 = 0.0 \text{ when } k_1 \leq 0.0$$
$$= 1.0 \text{ when } 0.0 < k_1$$

A_1, A_2, B_1, B_2, C_2 = regression coefficients to be estimated.

For definitions of other abbreviations, see abbreviations list (p. iii), main text.

Conditions:

$$Y_1 = Y_2 \text{ when } X = k_1$$

Final Equation Form

$$\begin{aligned} Y^* = 1.0 - X &+ I_2\Big(X + I_1 \{[(X - 1.0)/(k_1 - 1.0)] - 1.0\}\Big) \\ &+ A_1 I_2\Big(X + I_1 \{k_1 [(X - 1.0)/(k_1 - 1.0)] - X\}\Big) \\ &+ A_2 I_2\Big(X^2 + I_1 \{k_1^2 [(X - 1.0)/(k_1 - 1.0)] - X^2\}\Big) \\ &+ B_2 I_2 I_1 (k_1 - X)(1.0 - X) \\ &+ C_2 (X - 1.0)(X - I_2 X) \end{aligned}$$

APPENDIX C —

Final Equations for Predicting Diameter Inside Bark, Merchantable Height, or Merchantable Volume Inside Bark Above or Below Breast Height: Descriptions, Equation Forms, Conditions, Regression Coefficients

ABOVE BREAST HEIGHT

EQUATIONS [14] AND [14b]: DIAMETER INSIDE BARK ABOVE BREAST HEIGHT TO ANY HEIGHT

Description

These equations are equal to Equation [5] when $k_1 > 0.0$. Otherwise, they reduce to a single quadratic equation. The equations are further constrained such that the single quadratic and Equation [5] are equal when $k_1 = 0$.

Equation [14] uses CB, Equation [14b] $\widehat{CB}$.

Preliminary Equation Form

$$Y = Y_3 + I_2(Y_A - Y_3)$$

where:

$$Y_A = Y_1 + I_1(Y_2 - Y_1)$$
$$Y_1 = 1.0 + A_1 X + A_2 X^2$$
$$Y_2 = B_1 (X - 1.0) + B_2 (X^2 - 1.0)$$
$$Y_3 = 1.0 - X + C_2 (X^2 - X)$$
$$I_1 = 0.0 \text{ when } 0.0 \le X \le k_1$$
$$= 1.0 \text{ when } k_1 < X \le 1.0$$
$$I_2 = 0.0 \text{ when } k_1 \le 0.0$$
$$= 1.0 \text{ when } 0.0 < k_1$$
$$k_1 = (\alpha_1 CB - 4.5)/H$$
$$A_1 = b_{10} + b_{11}(H/DOB) + b_{12}(H/DOB)^2$$
$$A_2 = b_{20}$$

B_1, B_2, C_2 = coefficients eliminated in the final equation form because of the three conditions placed on the preliminary form.

b_{10}, b_{11}, b_{12}, b_{20}, α_1 = coefficients given in Table 3 of the main text.
For definitions of other abbreviations, see abbreviations list (p. iii), main text.

Conditions:

$$Y_1 = Y_2 \text{ and } \partial Y_1/\partial X = \partial Y_2/\partial X \text{ when } X = k_1$$
$$Y_A = Y_3 \text{ when } k_1 \le 0.0$$

Final Equation Form

$$\hat{Y} = 1.0 - X + I_2\Big(X + I_1 \{[(X - 1.0)/(k_1 - 1.0)] [1.0 + (k_1 - X)/(k_1 - 1.0)] - 1.0\}\Big) - (X - 1.0)(X - I_2 X) \quad Z1$$
$$+ A_1 I_2\Big(X + I_1 \{[(X - 1.0)/(k_1 - 1.0)] [X + k_1 (k_1 - X)/(k_1 - 1.0)] - X\}\Big) - (X - 1.0)(X - I_2 X) \quad Z2$$
$$+ A_2 I_2\Big(X^2 + I_1 \{k_1 [(X - 1.0)/(k_1 - 1.0)] [2X - k_1 + k_1 (k_1 - X)/(k_1 - 1.0)] - X^2\}\Big) \quad Z3$$

EQUATIONS [18] AND [18b]: MERCHANTABLE HEIGHT ABOVE BREAST HEIGHT TO ANY TOP DIAMETER INSIDE BARK

Merchantable height above breast height is estimated by:

$$\hat{h}_m = [-B + (B^2 + 4AC)^{1/2}]/2A$$
$$= [-B - (B^2 + 4AC)^{1/2}]/2A$$

Compute both roots of h_m. If one is < 0 or > H, then use the other root. If both roots meet these conditions, then use the larger of the two roots.

Equation [18] uses CB, Equation [18b] $\widehat{CB}$.

Conditions:

when:

$k_1 \leq 0.0$

$$A = -(A_1 + 1.0)/H^2$$
$$B = A_1/H$$
$$C = 1.0 - d_m/\widehat{DIB}$$

when:

$k_1 > 0.0$ and $d_m \geq d_{kh}$

$$A = A_2/H^2$$
$$B = A_1/H$$
$$C = 1.0 - d_m/\widehat{DIB}$$

when:

$k_1 > 0.0$ and $d_m \leq d_{kh}$

$$A = [A_2 k_1^2 - A_1 - 2 A_2 k_1 - 1.0]/[H^2 (k_1 - 1.0)^2]$$
$$B = [(2 k_1 - 1.0 + A_2 k_1^2 + A_1 k_1^2) - (A_2 k_1^2 - A_1 - 2 A_2 k_1 - 1.0)]/[H (k_1 - 1.0)^2]$$
$$C = -[d_m/\text{DIB} + (2 k_1 - 1.0 + A_2 k_1^2 + A_1 k_1^2)/(k_1 - 1.0)^2]$$

where:

$A_1 = b_{10} + b_{11}(H/DOB) + b_{12}(H/DOB)^2$

$A_2 = b_{20}$

$b_{10}, b_{11}, b_{12}, b_{20}, \alpha_1$ = coefficients given in Table 3 of the main text.

d_{kh} = diameter inside bark at the join point, predicted from Equations [14] and [14b].

For definitions of other abbreviations, see abbreviations list (p. iii), main text.

EQUATIONS [19] AND [19b]: MERCHANTABLE VOLUME ABOVE BREAST HEIGHT TO ANY TOP DIAMETER INSIDE BARK

Equation [19] uses CB, Equation [19b] $\widehat{CB}$.

<u>Preliminary Equation Form</u>

$$V_m = \int_{0.0}^{h_m} [\pi/576]\, d_m^{\,2}\, \partial h_i$$

<u>Final Equation Form</u>

when: $k_1 \leq 0.0$

$$\hat{V}_m = [\pi/576]\, \widehat{DIB}^2 \{h_m + (A_1/H)\, h_m^{\,2} - [2(A_1 + 1.0)/(3H^2)]\, h_m^{\,3} - [A_1(A_1 + 1.0)/(2H^3)]\, h_m^{\,4}$$
$$+ (A_1^{\,2}/3H^2)\, h_m^{\,3} + [(A_1 + 1.0)^2/(5H^4)]\, h_m^{\,5}\}$$

when: $k_1 > 0.0$ and $h_m > h_k$

$$V1 = [\pi/576]\, \widehat{DIB}^2\, h_k\, \{1.0 + (A_1/H)\, h_k + [(2A_2 + A_1^{\,2})/(3H^2)]\, h_k^{\,2}$$
$$+ [A_1 A_2/(2H^3)]\, h_k^{\,3} + [A_2^{\,2}/(5H^4)]\}\, h_k^{\,4}$$

$$V2 = \{\pi/[576\, (k_1 - 1.0)^4]\}\, \widehat{DIB}^2 \{(h_m - h_k)\, (P_1^{\,2}) + [h_m^{\,2} - h_k^{\,2}]\, [(-2P_1^{\,2} + 2P_1 P_2)/(2H)]$$
$$+ [h_m^{\,3} - h_k^{\,3}]\, [(P_1^{\,2} - 4P_1 P_2 + P_2^{\,2})/(3H^2)] + [h_m^{\,4} - h_k^{\,4}]\, [(2P_1 P_2 - 2P_2^{\,2})/(4H^3)]$$
$$+ [h_m^{\,5} - h_k^{\,5}]\, [P_2^{\,2}/(5H^4)]\}\}$$

$$\hat{V}_m = V1 + V2$$

when: $k_1 > 0.0$ and $h_m \leq h_k$

$$\hat{V}_m = \{[\pi/576]\, \widehat{DIB}^2\, h_m\, [1.0 + (A_1/H)\, h_m + [(2A_2 + A_1^{\,2})/(3H^2)]\, h_m^{\,2}$$
$$+ [A_1 A_2/(2H^3)]\, h_m^{\,3} + [A_2^{\,2}/(5H^4)]\, h_m^{\,4}\}$$

where:

$h_k = k_1 * H$

$A_1 = b_{10} + b_{11}\,(H/DOB) + b_{20}\,(H/DOB)^2$

$A_2 = b_{20}$

$b_{10}, b_{11}, b_{12}, b_{20}, \alpha_1$ = coefficients given in Table 3 of the main text

$P_1 = 2\,k_1 - 1.0 + A_2\,k_1^{\,2} + A_1\,k_1^{\,2}$

$P_2 = A_2\,k_1^{\,2} - A_1 - 2\,A_2\,k_1 - 1.0)$

∂h_i = differential of h_i

$\pi = 3.141592654$

For definitions of other abbreviations, see abbreviations list (p. iii), main text.

BELOW BREAST HEIGHT

EQUATION [20]: DIAMETER INSIDE BARK BELOW BREAST HEIGHT TO ANY STUMP HEIGHT
EQUATIONS [21] AND [21b]: DIAMETER INSIDE BARK 1.0 FOOT ABOVE THE GROUND

Description

Equation [20], based upon the equation for a neiloid frustum given in Husch et al. (1982), should only be used to estimate diameters below breast height.

Final Equation Form

$$\hat{Y}_{bbh} = \left(\{4.5 - (\widehat{DIB}/\hat{d}_{1.0})^{2/3} - bh_i[1.0 - (\widehat{DIB}/\hat{d}_{1.0})^{2/3}]\}/3.5\right)^{3/2} \qquad [20]$$

where:

$Y_{bbh} = bd_i/\hat{d}_{1.0}$

The value of $\hat{d}_{1.0}$ is obtained from one of the following two equations developed by Walters et al. (1985):

$$\hat{d}_{1.0} = g_0 + g_1 \, EXP[g_2 \, (H_t - CB)/H_t] \, DOB^{g_3} \qquad [21]$$

$$\hat{d}_{1.0} = f_0 + f_1 \, DOB^{f_2} \qquad [21b]$$

where:

$f_0, f_1, f_2, g_0, g_1, g_2, g_3$ = coefficients given in Table C-1.
Equation [21] can be used for Douglas-fir and grand/white fir if height to crown base is known; Equation [21b] can be used for all species or for Douglas-fir and grand/white fir when crown base is predicted.

bd_i = diameter inside bark at the i^{th} point of interest below breast height

bh_i = distance from ground to the i^{th} point of interest below breast height

π = 3.141592654

For definitions of other abbreviations, see abbreviations list (p. iii), main text.

TABLE C-1.

REGRESSION COEFFICIENTS FOR PREDICTING STUMP DIAMETER INSIDE BARK BELOW BREAST HEIGHT, BY SPECIES, EQUATIONS [21b] AND [21].

	Regression coefficients						
Species	f_0	f_1	f_2	g_0	g_1	g_2	g_3
		Equation [21b]			Equation [21]		
Douglas-fir	0.000000	0.989819	1.000000	0.000000	0.938343	0.101792	1.000000
Grand/white fir	0.287414	0.828652	1.082631	0.341157	0.753147	0.101138	1.0952985
Ponderosa pine	0.000000	1.000000	1.000000	NA	NA	NA	NA
Sugar pine	0.000000	1.039080	1.000000	NA	NA	NA	NA
Incense-cedar	0.476734	0.819613	1.067437	NA	NA	NA	NA

NA = Not applicable.

EQUATIONS [22] AND [22b]: MERCHANTABLE VOLUME INSIDE BARK BELOW BREAST HEIGHT TO ANY STUMP HEIGHT

Preliminary Equation Form

$$V_{bbh} = \int_{h_S}^{4.5} (\pi/576) * bd_i^2 \partial L$$

Final Equation Form

$$\hat{V}_{bbh} = (W_1/43904)(729.0 + 81.0\,W_2 + 297.0\,W_2^2 + 265.0\,W_2^3)$$
$$+ (W_1/6174)(W_3^3 h_S - 1.5\,W_3^2\,W_4\,h_S^2 + W_3\,W_4^2\,h_S^3 - 0.25\,W_4^3\,h_S^4)$$

where:

$W_1 = \pi\,(0.25\,\hat{d}_{1.0}^2)$

$W_2 = (\widehat{DIB}/\hat{d}_{1.0})^{2/3}$

$W_3 = 4.5 - W_2$

$W_4 = 1.0 - W_2$

V_{bbh} = volume below breast height to any stump height

(The value of bd_i is obtained from Equation [20]. The value of $\hat{d}_{1.0}$ is obtained from one of two equations developed by Walters et al. (1985); see Equations [21] and [21b], page 37.)

∂L = differential of L

L = distance from stump height to breast height

π = 3.141592654

For definitions of other abbreviations, see abbreviations list (p. iii).

APPENDIX D —

Growth and Yield Sample and Felled-Tree Subsample: Distribution by Species across Height/Diameter (H/DOB) Ratio and Crown Ratio Classes

TABLE D-1.

DISTRIBUTION ACROSS H/DOB RATIO AND CROWN RATIO CLASSES FOR THE DOUGLAS-FIR GROWTH AND YIELD SAMPLE. THE OUTLINED PORTION SHOWS THE DISTRIBUTION OF THE FELLED-TREE SUBSAMPLE.

	Crown Ratio											
H/DOB	.00	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00	Total
.00	--	2	5	7	4	2	4	2	4	1		31
1.00	--	2	3	8	4	4	1	4	7	3	1	37
2.00	--	7	17	16	24	28	12	9	11	7		131
3.00	3	19	36	58	78	86	93	29	39	21	2	464
4.00	2	40	77	169	304	377	296	200	145	127	18	1755
5.00	2	38	156	439	853	901	582	314	279	234	25	3823
6.00	2	53	263	637	966	844	492	247	196	122	9	3831
7.00		66	293	630	649	428	271	156	74	26	4	2597
8.00	5	61	273	353	290	229	99	59	31	6		1406
9.00	2	56	167	187	118	71	49	28	10	3	1	692
10.00	3	35	103	88	54	25	14	5	3	--	--	330
11.00		18	43	36	16	18	5	2		--	--	138
12.00	2	5	14	23	11	8	1	3	1	--	--	68
13.00	--	4	7	6	5		3	1	1	--	--	27
14.00	--	--		1	1	2	1	--	--		--	5
15.00	--		1		--	--	--	--	--	1	--	2
16.00	--	1	--	1	--	--	--	--	--	--	--	2
17.00	--		--	--	--	--	--	--	--	--	--	
18.00	--	1	--	--	--	--	--	--	--	--	--	1
19.00	--	--	--	--	--	--	--	--	--	--	--	
20.00	--	--	--	--	--	--	--	--	--	--	--	
Total	21	408	1458	2660	3377	3023	1923	1059	800	551	60	15340

TABLE D-2.

DISTRIBUTION ACROSS H/DOB RATIO AND CROWN RATIO CLASSES FOR THE GRAND FIR GROWTH AND YIELD SAMPLE. THE OUTLINED PORTION SHOWS THE DISTRIBUTION OF THE FELLED-TREE SUBSAMPLE.

	Crown Ratio											
H/DOB	.00	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00	Total
.00	--					1	1	2	1	--	--	5
1.00	--	3	1	1	1	2	1	1	2	--	--	12
2.00	--	5	5	5	4	4	2	2	4			31
3.00	--	7	6	11	10	7	7	9	19	6	1	83
4.00	--	4	13	21	24	21	22	34	25	24	5	193
5.00	--	7	16	30	59	67	45	35	33	25	2	319
6.00		10	17	25	61	60	52	34	21	6	1	287
7.00	2	7	13	29	34	35	29	14	10	4	--	177
8.00		3	7	19	18	16	8	2	1	1	--	75
9.00	1	10	9	14	13	2	2	2		--	--	53
10.00	2	5	5	5	1	--	1	1	1	--	--	21
11.00	--	--	--	--	--	--	1	--	--	--	--	1
12.00	--	--	--	--	--	--	--	--	--	--	--	
13.00	--	--	--	--	--	--	--	--	--	--	--	
14.00	--	--	--	--	--		--	--	--	--	--	
15.00	--	--	--	--	--	1	--	--	--	--	--	1
16.00	--	--	--	--	--	--	--	--	--	--	--	
17.00	--	--	--	--	--	--	--	--	--	--	--	
18.00	--	--	--	--	--	--	--	--	--	--	--	
19.00	--	--	--	--	--	--	--	--	--	--	--	
20.00	--	--	--	--	--	--	--	--	--	--	--	
Total	5	61	92	160	225	217	171	136	117	66	9	1259

TABLE D-3.

DISTRIBUTION ACROSS H/DOB RATIO AND CROWN RATIO CLASSES FOR THE WHITE FIR GROWTH AND YIELD SAMPLE. THE OUTLINED PORTION SHOWS THE DISTRIBUTION OF THE FELLED-TREE SUBSAMPLE.

	Crown Ratio											
H/DOB	.00	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00	Total
.00	--	--	--	2	--	--	--	--	--	--	--	2
1.00	--	--	2	--	--	--	1	1	--	--	--	4
2.00	1	2	7	3	--	1	2	4	2	--	--	22
3.00	--	2	6	8	6	4	6	7	4	7	1	51
4.00	--	7	12	22	38	34	32	19	18	18	4	204
5.00	--	8	18	46	80	83	57	53	44	29	2	420
6.00	--	6	21	37	116	110	52	23	17	18	--	400
7.00	--	7	32	44	51	35	21	18	7	4	1	220
8.00	--	8	14	23	29	16	7	1	3	1	--	102
9.00	--	4	6	14	10	4	3	1	--	--	--	42
10.00	--	3	5	5	3	1	--	--	--	--	--	17
11.00	1	2	--	1	2	--	1	--	--	--	--	7
12.00	--	--	--	1	--	--	--	--	--	--	--	1
13.00	--	--	--	--	--	--	--	--	--	--	--	
14.00	--	--	--	--	--	--	--	--	--	--	--	
15.00	--	--	--	--	--	--	--	--	--	--	--	
16.00	--	--	--	--	--	--	--	--	--	--	--	
17.00	--	--	--	--	--	--	--	--	--	--	--	
18.00	--	--	--	--	--	--	--	--	--	--	--	
19.00	--	--	--	--	--	--	--	--	--	--	--	
20.00	--	--	--	--	--	--	--	--	--	--	--	
Total	2	49	123	206	335	288	182	127	95	77	8	1492

TABLE D-4.

DISTRIBUTION ACROSS H/DOB RATIO AND CROWN RATIO CLASSES FOR THE PONDEROSA PINE GROWTH AND YIELD SAMPLE. THE OUTLINED PORTION SHOWS THE DISTRIBUTION OF THE FELLED-TREE SUBSAMPLE.

	Crown Ratio											
H/DOB	.00	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00	Total
.00	--	--	--	--	1	--	--	--	--	--	--	1
1.00	--	--	1	--	--	1	2	--	--	--	--	4
2.00	--	1	3	1	2	2	5	5	12	7	--	38
3.00	--	4	5	18	29	30	33	27	32	17	2	197
4.00	1	9	15	35	100	108	74	34	18	3	--	397
5.00	1	5	31	79	186	147	40	17	5	1	--	512
6.00	--	8	24	76	118	44	14	4	1	--	--	289
7.00	2	9	32	49	52	14	1	1	--	--	--	160
8.00	1	4	26	35	14	7	1	--	--	1	--	89
9.00	--	7	9	16	5	--	--	--	1	--	--	38
10.00	--	4	10	7	4	--	1	--	--	1	--	27
11.00	1	4	7	1	--	--	1	--	--	--	--	14
12.00	--	1	1	--	--	--	--	--	--	--	--	2
13.00	--	--	--	--	--	--	--	--	--	--	--	
14.00	--	--	1	--	--	--	--	--	--	--	--	1
15.00	1	--	--	--	--	--	--	--	--	--	--	1
16.00	--	--	--	--	--	--	--	--	--	--	--	
17.00	--	--	--	--	--	--	--	--	--	--	--	
18.00	--	--	--	--	--	--	--	--	--	--	--	
19.00	--	--	--	--	--	--	--	--	--	--	--	
20.00	--	--	--	--	--	--	--	--	--	--	--	
Total	7	56	165	317	512	353	172	88	68	30	2	1770

TABLE D-5.

DISTRIBUTION ACROSS H/DOB RATIO AND CROWN RATIO CLASSES FOR THE SUGAR PINE GROWTH AND YIELD SAMPLE. THE OUTLINED PORTION SHOWS THE DISTRIBUTION OF THE FELLED-TREE SUBSAMPLE.

H/DOB	Crown Ratio .00	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00	Total
.00	--	--	--	1	--	--	--	--	--	--	--	1
1.00	--	--	1	--	--	--	1	--	1	--	--	3
2.00	--	1	1	1	--	3	--	--	1	--	--	7
3.00	--	1	3	4	13	16	14	2	4	2	--	59
4.00	--	1	10	16	30	36	31	19	9	9	--	161
5.00	--	1	4	12	29	27	14	9	7	1	--	104
6.00	--	2	5	12	12	15	1	3	--	1	--	51
7.00	--	--	1	6	6	--	--	--	--	--	--	13
8.00	--	1	1	2	2	1	--	--	--	--	--	7
9.00	--	--	--	--	--	--	--	--	--	--	--	
10.00	--	--	--	--	--	1	--	--	--	--	--	1
11.00	--	--	1	--	--	--	--	--	--	--	--	1
12.00	--	--	--	--	--	--	--	--	--	--	--	
13.00	--	--	--	--	--	--	--	--	--	--	--	
14.00	--	--	--	--	--	--	--	--	--	--	--	
15.00	--	--	--	--	--	--	--	--	--	--	--	
16.00	--	--	--	--	--	--	--	--	--	--	--	
17.00	--	--	--	--	--	--	--	--	--	--	--	
18.00	--	--	--	--	--	--	--	--	--	--	--	
19.00	--	--	--	--	--	--	--	--	--	--	--	
20.00	--	--	--	--	--	--	--	--	--	--	--	
Total		7	27	54	92	99	61	33	22	13		408

TABLE D-6.

DISTRIBUTION ACROSS H/DOB RATIO AND CROWN RATIO CLASSES FOR THE INCENSE-CEDAR GROWTH AND YIELD SAMPLE. THE OUTLINED PORTION SHOWS THE DISTRIBUTION OF THE FELLED-TREE SUBSAMPLE.

H/DOB	Crown Ratio .00	.10	.20	.30	.40	.50	.60	.70	.80	.90	1.00	Total
.00	--	--	2	2	--	--	2	1	--	1	--	8
1.00	1	3	2	4	1	2	1	2	6	1	--	23
2.00	--	11	18	28	15	6	6	12	15	13	1	125
3.00	3	20	46	43	56	40	56	67	58	37	4	430
4.00	1	36	60	72	78	76	72	79	42	25	4	545
5.00	1	38	61	64	59	47	36	32	8	5	--	351
6.00	--	19	35	20	22	9	9	6	6	1	--	127
7.00	--	10	6	11	5	6	--	2	1	--	--	41
8.00	--	2	4	1	1	1	--	1	2	--	--	12
9.00	--	1	2	--	2	--	--	--	1	--	--	6
10.00	--	--	--	--	1	1	--	--	--	--	--	2
11.00	--	1	--	--	--	--	1	--	--	1	--	3
12.00	--	--	--	--	--	--	1	--	1	--	--	2
13.00	--	--	--	--	--	--	--	--	--	--	--	
14.00	--	--	--	--	--	--	--	--	--	--	--	
15.00	--	--	--	--	--	--	--	--	--	--	--	
16.00	--	--	--	--	--	--	--	--	--	--	--	
17.00	--	--	--	--	--	--	--	--	--	--	--	
18.00	--	--	--	--	--	--	--	--	--	--	--	
19.00	--	--	--	--	--	--	--	--	--	--	--	
20.00	--	--	--	--	--	--	--	--	--	--	--	
Total	6	141	236	244	240	189	184	202	140	84	9	1675

WALTERS, D.K., and D.W. HANN. 1986. TAPER EQUATIONS FOR SIX CONIFER SPECIES IN SOUTHWEST OREGON. Forest Research Laboratory, Oregon State University, Corvallis. Research Bulletin 56. 41 p.

Taper equations predicting upper stem diameters inside bark are presented for Douglas-fir, grand fir, white fir, ponderosa pine, sugar pine, and incense-cedar, the six most common species in the mixed conifer zone of southwest Oregon. Fourteen different equations, including 11 segmented polynomial equations, are examined in a preliminary analysis so that the most appropriate form can be identified. The best choice is further modified and any undesirable equation behavior eliminated. Because height to crown base significantly improves the model, an equation predicting height to live-crown base also is developed for foresters lacking actual crown measurements. The final taper equation may be manipulated to yield estimates of merchantable height and volume to any top diameter as well. A summary chart shows how to apply the final equations, which predict diameter inside bark above breast height to any height, merchantable height, and merchantable volume inside bark above breast height to any top diameter inside bark, as well as other equations which predict diameter and merchantable volume inside bark below breast height to any stump height.

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